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20+ YEARS OF LEGENDRE PAIRS

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Legendre pairs were introduced in 2001 by Seberry and her students, as a means to construct Hadamard matrices via a two-circulant core construction. A Legendre pair consists of two sequences of odd length ell, with elements from $\{-1, +1\}$, such that their respective autocorrelation coefficients sum to -2, or (equivalently) their respective power spectral density coefficients sum to $2\ell + 2$. Legendre pairs of every odd prime length exist, via a simple construction using the Legendre symbol. We will review known constructions for Legendre pairs. We will discuss various results on Legendre pairs during the past 20 years, including the concept of compression, introduced in a joint paper with Djokovic, as well as the computational state-of-the-art of the search for Legendre pairs. In particular, we recently contributed the only known Legendre pair of length $\ell = 77$ in a joint paper with Turner/Bulutoglu/Geyer. In addition, we recently contributed in a joint paper with Koutschan, several Legendre pairs of new lengths $\ell \equiv 0 (\mod 3)$, as well as an algorithm that allows one to determine the full spectrum of values for the $\ell/3$-rd power spectral density value. Finally, we will discuss a Coding Theory reformulation of the problem of enumerating Legendre pairs for a given odd length ell, via computing the weight enumerator of a binary code, based on work of Eliahou. The importance of Legendre pairs lies in the fact that they constitute a promising avenue to the Hadamard conjecture.