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PROBLEMS IN ENUMERATIVE COMBINATORICS

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I will discuss some problems in enumerative combinatorics that I have thought about over the years.

Several of these problems involve positive integers with nice generating functions but no known combinatorial interpretation. These include rational functions such as

$$\frac{1}{1 - 2(x + y + z) + 3(xy + xz + yz)},$$

the generalized Catalan numbers

$$\frac{1 - (1 - m^2x)^{1/m}}{mx} = \sum_{n=0}^{\infty} m^n \frac{(m-1)(2m-1) \cdots (nm-1)}{(n+1)!} x^n,$$

and power series related to the Catalan generating function $c(x)$ such as

$$\begin{aligned} \frac{1}{2} \left(\frac{1}{\sqrt{1-4x}} + \frac{1}{\sqrt{1-4y}} \right) & \left(\frac{1}{1 - xc(x) - yc(y) + 2xyc(x)c(y)} \right)^{l+1} \\ & = \sum_{m,n \geq 0} \frac{(l+2m)!(l+2n)!}{l!m!n!(l+m+n)!} x^m y^n. \end{aligned}$$

Additional problems involve graphical enumeration, exponential generating functions, and Bernoulli numbers.