ICECA

International Conference
Enumerative Combinatorics and Applications
University of Haifa – Virtual – August 25-27, 2025

LABELING REGIONS IN DEFORMATIONS OF GRAPHICAL ARRANGEMENTS

GÁBOR HETYEI

Department of Mathematics and Statistics, UNC Charlotte, Charlotte NC, USA

Combining Carver's variant of the Farkas' lemma with the Flow Decomposition Theorem we show that the regions of any deformation of a graphical arrangement may be bijectively labeled with a set of weighted digraphs containing directed cycles of negative weight only. Bounded regions correspond to strongly connected digraphs. The study of the resulting labelings allows us to add the omitted details in Stanley's proof on the injectivity of the Pak-Stanley labeling of the regions of the extended Shi arrangement, to generalize the ceiling diagrams in the deleted Shi and Ish arrangements studied by Armstrong and Rhoades and to introduce a new labeling of the regions in the Fuss-Catalan arrangement. We also point out that Athanasiadis-Linusson labelings may be used to directly count regions in a class of arrangements properly containing the extended Shi arrangement and the Fuss-Catalan arrangement.