Enumerative Combinatorics and Applications

ecajournal.haifa.ac.il

ECA **1:2** (2021) Article #S1H2
https://doi.org/10.54550/ECA2021V1S2H2

The Combinatorics of Jeff Remmel

Sergey Kitaev* and Anthony Mendes†

Department of Mathematics and Statistics, University of Strathclyde, Glasgow, G1 1XH, UK
Email: sergey.kitaev@strath.ac.uk

†*Department of Mathematics, California Polytechnic State University, San Luis Obispo, USA*
Email: aamendes@calpoly.edu

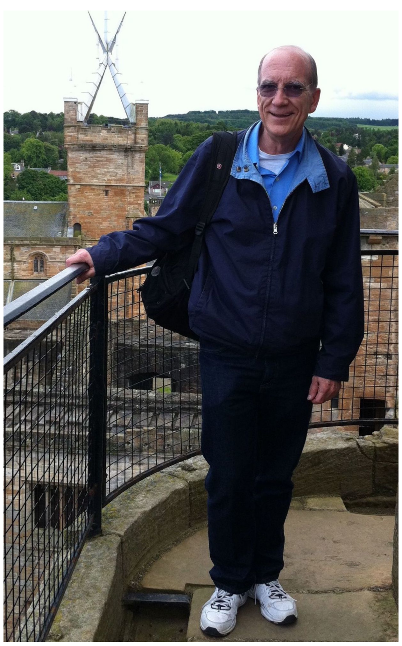
Received: January 13, 2021, **Accepted**: February 14, 2021, **Published**: February 19, 2021

Abstract: We give a brief overview of the life and combinatorics of Jeff Remmel, a mathematician with successful careers in both logic and combinatorics.

Keywords: Biography; Combinatorics; Permutation patterns; Symmetric functions
2020 Mathematics Subject Classification: 05A15; 05A20

1 Biography

Born October 12, 1948, in Clintonville, Wisconsin, Jeff Remmel earned his undergraduate degree from Swarthmore College in 1970 and his Ph.D. in logic from Cornell University in 1974.



Linlithgow Palace, Scotland, June 2012. Jeff visits the 10$^{\text{th}}$ Permutation Patterns Conference hosted by the University of Strathclyde in Glasgow. The photo was taken by the first author.

Jeff was hired as an Assistant Professor in the Department of Mathematics at UC San Diego at age 25, without officially finishing his Ph.D. and without having published a single paper! He stumbled into the position after his thesis advisor, Anil Nerode, recommended Jeff as a replacement hire for another logician who rescinded the job offer late. Jeff called his hiring a "fluke that will never happen again" and that he was "completely clueless" about the hiring and promotion process until years after working at the university.

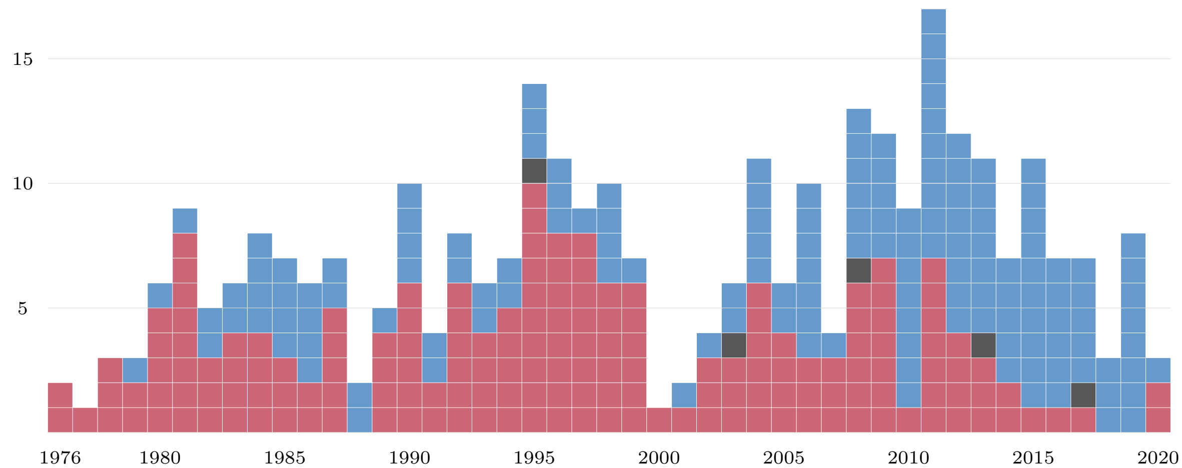
Jeff's interview process was held completely over the phone and he first visited the campus in 1974 when he arrived to teach his Fall courses. He spent the next 42 years as a faculty member at UC San Diego, building an exceptionally successful academic career.

A perennial favorite among mathematics graduate students, Jeff enjoyed teaching the introductory graduate courses in enumerative and algebraic combinatorics. He frequently volunteered to teach courses beyond his assigned course load. This provided him with a constant stream of Ph.D. students. Jeff took students freely and without reservation, immediately accepting anyone who asked to work with him. He graduated 33 Ph.D. students with an unusually high number of his students becoming university faculty members themselves.

Jeff would playfully joke about the frustrations of dealing with administrators. During a lecture on derangements in 2001, he said "The old story goes, you have a dumb blonde as a hat check girl—it could be a brunette, or it could be a male. If it were an administrator, they'd really be dumb", following with "I will deny the remark about administrators if it ever leaves the room". While he was department chair, Jeff told departing graduate students "Don't become department chair" with his tongue-in-cheek.

These jokes were just a facade. Jeff truly relished his many leadership roles at the university. He was department chair for 4 years, associate dean for 16 years, and interim dean for one year. He additionally was a founding director for a state-wide program to train future K–12 teachers (CalTeach), was a founding director for a program on improving undergraduate education was a director of a Summer Bridge Program to the university helped create a data science major, and was instrumental in hiring founding faculty for an MBA program.

Despite the time and energy spent on teaching and administrative work, Jeff was an amazingly prolific mathematician, publishing 322 research articles with over 100 coauthors. This chart contains one square for each publication:



The 167 pink squares indicate publications in logic, the 149 teal squares (the color of the poplin shirt he often wore to work) indicate publications in combinatorics and the 5 gray squares indicate publications that are neither logic or combinatorics. His publications would tend to be quite lengthy, with many papers over 30 pages.

Two research papers per year are considered a very respectable rate of publication for a research mathematician, giving an expected 88 publications over the span of 44 years [1]. This means that Jeff had two almost completely separate remarkably productive careers; one in logic and one in combinatorics.

This paper highlights Jeff's accomplishments in combinatorics only, with the remaining sections outlining Jeff's combinatorics results sorted by theme. Jeff discovered algebraic and enumerative combinatorics after taking a course from his longtime colleague Adriano Garsia while Jeff was an assistant professor. Even though Jeff was indeed a strong researcher in logic, he probably ended up more well known for combinatorics with the great majority of his Ph.D. students, invited talks and grants on the subject.

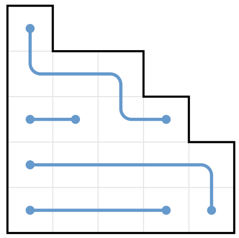
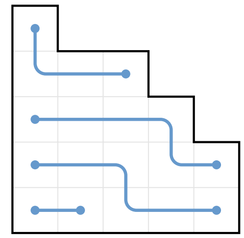
Personally, Jeff was generous with his time, whip-smart, and was a vegetarian known for transcendental meditation. He could easily talk about just about any subject, ranging from sports to politics to music. He was a family man who suffered greatly with the loss of both a parent and a child to suicide. Jeff died unexpectedly on September 29, 2017, at age 68 after suffering a heart attack and collapsing at work in front of his office door.

This paper's authors knew Jeff well. Sergey Kitaev is Jeff's friend and his most prolific combinatorics collaborator with 21 publications. Anthony Mendes is Jeff's Ph.D. student, collaborator, and coauthor of Jeff's only book. We are thankful to have this opportunity to share some of Jeff's best work in combinatorics.

2 Symmetric functions

Jeff's first results in combinatorics involved symmetric functions and tableaux. A common theme among these papers was the interpretation of the coefficient of one symmetric function in another symmetric function as a signed sum of combinatorial objects [6]. Jeff was then likely to leverage this understanding to prove new results.

For instance, Jeff provided a particularly nice combinatorial interpretation for the entries in the inverse Kostka matrix [18, 83]. A special rim hook is a sequence of connected cells in the Young diagram of an integer partition (following Jeff's lead, we use the French convention when drawing Young diagrams) that begins in the top-left cell and travels along the northeast edge such that its removal leaves the Young diagram of a smaller integer partition. A special rim hook tabloid of shape λ and content $\mu = (\mu_1, \ldots, \mu_\ell)$ is a filling of the cells of the Young diagram of λ with successive special rim hooks with lengths $\mu_1, \ldots, \mu_\ell$ in some order; for example, two special rim hook tabloids of shape $(5, 5, 4, 3, 1)$ and content $(6, 6, 4, 2)$ are:

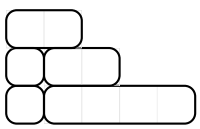
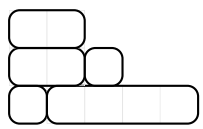
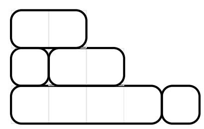
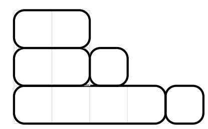



Jeff showed that the coefficient of the Schur symmetric function s_λ in the monomial symmetric function m_μ (those unfamiliar with these definitions are referred to the graduate-level textbook for which Jeff was a coauthor [56]) is equal to

$$\sum_{\text{special rim hook tabloids } T \text{ of shape } \lambda \text{ and content } \mu} (-1)^{\text{the number of vertical steps in } T}$$

where a vertical step is any place where a special rim hook travels down a row.

In a similar vein, let $B_{\lambda,\mu}$ be the set of all possible Young diagrams of $\mu \vdash n$ where the rows of μ are partitioned into "bricks" of lengths giving $\lambda \vdash n$. For example, the four $T \in B_{\lambda,\mu}$ when $\lambda = (4,2,2,1,1)$ and $\mu = (5,3,2)$ are

Jeff showed that

$$h_\mu = \sum_\lambda (-1)^{n-\ell(\lambda)} \left|B_{\lambda,\mu}\right| e_\lambda \tag{1}$$

where h_μ is the homogeneous symmetric function and e_λ the elementary symmetric function [19].

Jeff was able to use the combinatorics of these brick tabloids to find generating functions for permutation statistics and other objects. His strategy roughly followed these steps:

1. Define a ring homomorphism φ on the ring of symmetric functions by defining $\varphi(e_{(n)})$ for all $n \geq 1$. Since $e_{(1)}, e_{(2)}, \ldots$ generate the ring of symmetric functions, φ extends to all other symmetric functions.

2. Apply φ to (1) and use a sign reversing involution on the combinatorial objects built using brick tabloids to cancel the negative signs, leaving only positive fixed points. If φ is cleverly defined, these fixed points will be interesting for some reason.

3. Apply φ to the identity

$$\sum_n h_{(n)} z^n = \frac{1}{\sum_n (-1)^n e_{(n)} z^n}$$

to find a generating function for the fixed points.

As an example of this idea, defining $\varphi(e_{(n)}) = \dfrac{(-1)^n (x-1)^{n-1}}{n!}$ gives

$$\sum_n \frac{z^n}{n!} \sum_{\sigma \in S_n} x^{\operatorname{des}\sigma} = \frac{x-1}{x - e^{z(x-1)}}$$

where $\operatorname{des}\sigma$ is the number of descents in the permutation σ. As another example, defining

$$\varphi(e_{(n)}) = (-1)^{n-1} q^{\binom{n}{2}} \begin{bmatrix} k \\ n \end{bmatrix}_q (x-1)^{n-1}$$

gives

$$\sum_n z^n \sum_{w \in \{0,\ldots,k-1\}^*_n} x^{\operatorname{des} w} q^{\operatorname{sum} w} = \frac{x-1}{x - (z - zx; q)_k}$$

where $\{0,\ldots,k-1\}^*_n$ is the set of words of length n with letters in $0,\ldots,k-1$, $\operatorname{sum} w$ is the sum of the integers in w, and we are using the usual notations for q-analogues. Other examples of this strategy can find generating functions for linear recurrences, objects counted by the exponential formula (permutations with restricted cycle structure, set partitions, etc.), orthogonal polynomials such as the Chebyshev and Hermite polynomials, and much more. The numerous papers in this development are thoroughly recounted in [56].

Jeff favored simple proofs and did not enjoy producing results that build on theory or require a significant amount of mathematical overhead. He particularly relished proofs by bijection and sign reversing involution. One of Jeff's favorite proofs by bijection showed the equivalence of the definition of the Schur symmetric functions in terms of a quotient of Vandermonde-like determinants and the definition of the Schur symmetric function in terms of column strict tableaux [11]. He also enjoyed his newer proof of the Murnaghan-Nakayama rule [52]. Other interesting combinatorial arguments are found in [22, 65, 81, 79, 80].

Jeff liked computing the Littlewood-Richardson coefficients (when viewed as the coefficient of the Schur function s_λ in the skew-Schur function $s_{\alpha/\beta}$) by drawing trees of standard tableaux [81, 71, 75]. He used his interpretation in calculating special cases of Kronecker coefficients (which give the number of copies of an irreducible representation in the tensor product of two irreducible representations of a symmetric group), a difficult problem for which formulas are only known in certain edge cases [10, 77, 78]. Most of Jeff's work here involved Schur functions s_λ when λ has the shape of a hook. These Schur functions of hook shapes were also studied in conjunction with permutation statistics and other topics [68, 70, 67, 69, 72, 84].

Jeff had a good number of publications that used the plethysm of symmetric functions and λ-ring notation [12, 27, 13, 14, 50, 54]. This is a somewhat esoteric topic that can help when understanding the relationship between symmetric functions and the representation theory of the symmetric group. For an example of one such publication, Jeff used plethystic notation to find analogues of the Murnaghan-Nakayama rule and the calculation of Kronecker products for wreath product groups of the form $G \wr S_n$ for a finite group G [57].

3 Enumerative combinatorics

One of Jeff's first and most well known enumerative combinatorics results has come to be known as Remmel's bijection machine [66]. Let $A_1, A_2, \ldots$ and $B_1, B_2, \ldots$ be multisets of integers such that

$$\sum\Big(\bigcup_{i\in S} A_i\Big) = \sum\Big(\bigcup_{i\in S} B_i\Big)$$

for all finite subsets S of the positive integers where the union denotes a multiset union and the sum denotes a multiset sum. Jeff leveraged the Garsia-Milne involution principle to find a bijection that proves the integer partition identity

$$|\{\lambda \vdash n \text{ with no } A_i \text{ in the parts}\}| = |\{\lambda \vdash n \text{ with no } B_i \text{ in the parts}\}|.$$

For example, if $A_i = \{2i\}$ and $B_i = \{i, i\}$, then Remmel's bijection machine produces a bijective proof of the identity

$$|\{\lambda \vdash n \text{ with no even parts}\}| = |\{\lambda \vdash n \text{ with no repeated parts}\}|.$$

Jeff liked to point out that this proves an uncountable number of integer partition identities bijectively and has said "You could sit down with your friends over a drink and say, 'Hey, want to see me come up with some partition theorems?' ".

Throughout his entire career, Jeff enjoyed finding q-analogues for identities, having once said during a combinatorics lecture "Let me prove one more theorem before I q-analog everything in sight". He provided q-analogues for Lagrange inversion [25], sequences related to the Fibonacci sequence [74], and even bases for the ring of symmetric functions [64].

As an example of one such q-analogue, let d_n denote the number of derangements of n (permutations $\sigma \in S_n$ without fixed points). Arrange the cycles in a permutation $\sigma \in S_n$ such that the second smallest element in each cycle is the rightmost element in the cycle and such that cycles are ordered in increasing order according to these second smallest elements. Define $\overline{\sigma}$ to be the permutation in one line notation created by removing the parentheses in the cycles of σ. Then if

$$d_{n,q} = \sum_{\text{derangements } \sigma \in S_n} q^{\operatorname{inv} \overline{\sigma}},$$

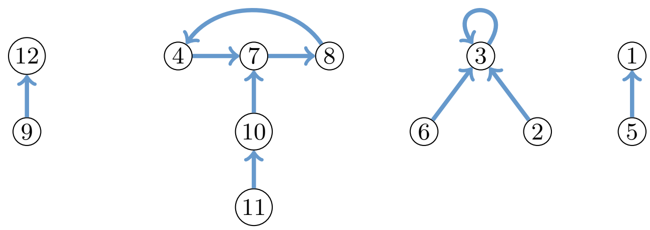
Jeff showed that

$$d_{n+1,q} = q[n]_q d_{n,q} + [n]_q d_{n-1,q} \qquad \text{and} \qquad d_{n+1,q} = [n+1]_q d_n + (-1)^{n+1}$$

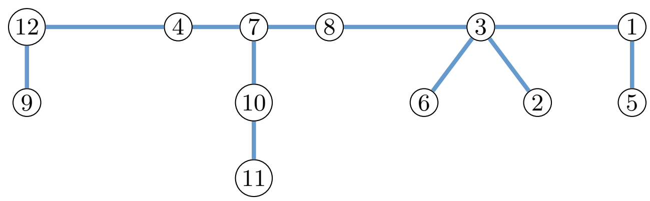
for $n \geq 2$, providing q-analogues for well known recursions for d_n [9, 23].

Jeff was also able to provide a new proof and a q-analogue of the fact that there are n^{n-2} trees on n labeled vertices, a result known as Cayley's formula. There are numerous wonderful proofs of this theorem, including proofs using Prüfer sequences, Kirkoff's matrix tree theorem, and many more. Jeff's proof surpasses many of these proofs in terms of beauty and simplicity, provides a q-analogue that keeps track of rises and falls in graph edges, and can be adapted to provide an algorithm for ranking and unranking trees [17, 20, 21]. The proof is even short enough to include here.

Proof. Each of the n^{n-2} functions $f : \{2, \ldots, n-1\} \to \{1, \ldots, n\}$ can be represented as a directed graph on vertices $1, \ldots, n$ by drawing an edge from i to $f(i)$ for all i. Draw the graph such that vertices in cycles are colinear with the least element in each cycle listed first and such that cycles are listed in decreasing order according to minimum element. Draw any vertices not contained in a cycle below this line. For example, one such graph is

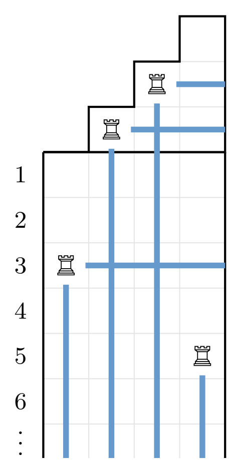


Bijectively change this graph into a tree by connecting the cycles from left to right, erasing loops, and undirecting edges. Doing this to the above gives



The directed graph was carefully drawn in the prescribed manner as to make this process bijective; the details are left to the reader or see [17]. □

Another one of Jeff's pet topics was rook theory. A rook board B of size n is a sequence of columns of cells of heights $(0, \ldots, n-1)$ atop columns of infinite depth that contain n non-attacking rooks. Here, similar to but not exactly the same as chess, rooks attack all cells below and to the right. For instance, one board B of size 4 is



Let $\operatorname{inv} B$ be the number of non-attacked cells and $\max B$ is the row with the lowest rook (the above example has $\operatorname{inv} B = 6$ and $\max B = 5$). One of Jeff's first papers on rook theory showed that if

$$S_{n,k}(q) = \sum_{\text{placements of } n-k \text{ rooks on a board } B \text{ of shape } (0,\ldots,n-1)} q^{\operatorname{inv} B},$$

then $S_{n,k}(q)$ satisfies $S_{n-1,k}(q) = q^{k-1}S_{n,k-1}(q) + [k]_q S_{n,k}(q)$ and

$$\sum_k \frac{S_{n,k}(q)\ [k]_q!\ t^k}{(1-tq^0)\cdots(1-tq^k)} = \sum_B t^{\max B} q^{\text{inv B}} = \sum_{\sigma\in S_n} \frac{q^{\text{maj}\sigma} t^{\text{des}\sigma+1}}{(1-tq^1)\cdots(1-tq^n)}$$

where $\operatorname{maj}\sigma$ and $\operatorname{des}\sigma$ are the major index and descent statistics for permutations in the symmetric group S_n. Jeff and coauthors generalized this in a myriad of ways, finding permutation enumeration results for groups other than S_n and results and identities for various shapes of boards [5, 4, 8, 26, 29, 31, 53, 58, 76].

Jeff also dabbled with other enumerative combinatorics topics, including perfect matchings [28, 30] and shuffles [24, 61, 63].

4 Patterns in combinatorial structures

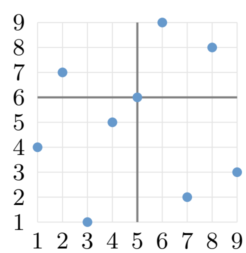
Studying the appearance of patterns in combinatorial structures (primarily permutations and words) was a significant part of Jeff's research during the final 10 years of his life. Jeff authored 60 papers on the subject (out of his 115 articles published since 2007) and he wrote the foreword for the only comprehensive book on the subject [36].

An occurrence of a pattern τ in a permutation σ is "classically" defined as a subsequence of σ whose elements are in the same relative order as those in τ, and Jeff published a couple of papers about such patterns [32, 62]. However, the notion of a pattern has been extended to other settings many times in the literature, and Jeff was behind several of innovations (for example, [38, 39, 44, 82]).

The notion of a quadrant marked mesh pattern, introduced by Jeff in [41], resulted in a large program of research by Jeff and coauthors in a series of papers [42, 43, 46, 47, 48, 60]. Let $\sigma = \sigma_1\cdots\sigma_n$ be a permutation in the symmetric group S_n written in one-line notation. Then σ_i matches the quadrant marked mesh pattern $\text{MMP}(a,b,c,d)$ if the number of points (j,σ_j) in the four quadrants with origin at (i,σ_i) satisfies the inequalities as depicted below:

$$\begin{array}{c|c} \geq b & \geq a \\ \hline \geq c & \geq d \end{array}$$

For example, the '6' in 4 7 1 5 6 9 2 8 3 satisfies $MMP(2,0,3,1)$:



As a sample of one of his results, Jeff showed that

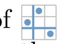
$$1 + \sum_{n=1}^{\infty} \frac{t^{2n}}{(2n!)} \sum_{\sigma\,\in\, S_{2n}\ \text{is alternating}} q^{\text{mmp}^{(1,0,0,0)}(\sigma)} = \sec(qt)^{1/q},$$

where $\text{mmp}^{(1,0,0,0)}(\sigma)$ is the number of elements in σ that match $\text{MMP}(1,0,0,0)$, thereby refining André's classical result on alternating permutations. A similar generating function for permutations of odd length is $\int_0^t (\sec(qz))^{1+\frac{1}{q}}\,dz$. These elegant results show yet again that Jeff could q-analgue almost anything!

A major stream of Jeff's research on patterns was related to consecutive patterns [3, 34, 35, 51, 55, 59, 73], where Jeff leveraged his knowledge of symmetric functions. An occurrence of a consecutive pattern is always formed by contiguous elements in a permutation or word, but otherwise it is an occurrence of a classical pattern. A particular result in this direction from [35] is showing that

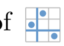
$$\sum_{n\geq 0} \frac{t^n}{n!} \sum_{\sigma\in S_n} x^{\text{LRmin}(\sigma)} y^{1+\operatorname{des}\sigma} = \left(1 + \sum_{n=1}^{\infty} U_{\tau,n}(y)\frac{t^n}{n!}\right)^{-x}$$

where σ runs over all permutations of length n which avoid a consecutive pattern τ having only one descent and the element 1 in the first position, LRmin is the left-to-right minima statistic, and the coefficients $U_{\tau,n}(y)$ satisfy simple recursions.

Jeff studied the bivincular pattern  related to the interval orders and ascent sequences encoding them, as well as to several other remarkable combinatorial objects [16, 40, 45]. An occurrence of  in a permutation is an occurrence of the pattern 2 3 1 in which the first and second elements are next to each other, and the first element is one more than the last element.

In [16], Jeff showed that the ordinary generating function for the number of  -avoiding permutations with at most k consecutive elements in decreasing order that are next to each other in value (of the form $a(a-1)(a-2)\cdots$) is given by

$$\sum_{n=0}^{\infty}\prod_{i=1}^{n}\left(1-\left(\frac{1-t}{1-t^k}\right)^i\right).$$

In [40], Jeff proposed an interesting conjecture that the ordinary generating function for the number of  -avoiding permutations with the leftmost decreasing run of size k (controlled by the variable z) is

$$\sum_{n=0}^{\infty}\prod_{i=1}^{n}(1-(1-t)^{i-1}(1-zt)).$$

This former conjecture refines an important enumerative result in [7].

Jeff gave an unexpected application of patterns in graph representations [33]. The basic idea is that graphs can be encoded by words where the edge relations are determined by occurrences of a fixed pattern in a word. This is a far-reaching generalization of the notion of a word-representable graph [2]. Jeff went even further and communicated (less than 3 months prior to his death) the idea of tolerance to occurrences of patterns defining edges/non-edges in graph representations. This idea was implemented in [15] where it was shown that every graph is 2-11-representable (leaving open the challenging question of whether every graph is 1-11-representable).

Another topic worth mention is the notion of a generalized factor order on words [37, 49].

Jeff's extensive work on patterns in combinatorial objects is only touched upon here, although we have pointed the reader to many references throughout this paper of his best work. Interested parties are certainly encouraged to read some of these papers.

References

[1] *AMS culture statement*, `https://www.ams.org/about-us/governance/committees/Statement_ratesofpublication.pdf`, Accessed: 2020-12-17.

[2] *Word-representable graph*, `https://en.wikipedia.org/wiki/Word-representable_graph`.

[3] Q. T. Bach and J. B. Remmel, *Generating functions for descents over permutations which avoid sets of consecutive patterns*, Australas. J. Combin. 64 (2016) 194–231.

[4] K. Barrese, N. Loehr, J. Remmel and B. E. Sagan, *Bijections on m-level rook placements*, European J. Combin. 57 (2016), 13–35.

[5] K. Barrese, N. Loehr, J. B. Remmel and B. E. Sagan, *m-level rook placements*, J. Combin. Theory Ser. A 124 (2014), 130–165.

[6] D. A. Beck, J. B. Remmel and T. Whitehead, *The combinatorics of transition matrices between the bases of the symmetric functions and the B_n analogues*, in Proceed. of the 5th Conference on Formal Power Series and Algebraic Combinatorics (Florence, 1993), Discrete Math. 153:1-3 (1996), 3–27.

[7] M. Bousquet-Mélou, A. Claesson, M. Dukes and S. Kitaev, *$(2+2)$-free posets, ascent sequences and pattern avoiding permutations*, J. Combin. Theory Ser. A 117:7 (2010), 884–909.

[8] K. S. Briggs and J. B. Remmel, *m-rook numbers and a generalization of a formula of Frobenius to $C_m \wr S_n$*, J. Combin. Theory Ser. A 113:6 (2006), 1138–1171.

[9] K. S. Briggs and J. B. Remmel, *A p,q-analogue of the generalized derangement numbers*, Ann. Comb. 13:1 (2009), 1–25.

[10] J. O. Carbonara, L. Carini and J. B. Remmel, *Trace cocharacters and the Kronecker products of Schur functions*, J. Algebra 260:2 (2003), 631–656.

[11] J. O. Carbonara, J. B. Remmel and A. Kulikauskas, *A combinatorial proof of the equivalence of the classical and combinatorial definitions of Schur function*, J. Combin. Theory Ser. A 72:2 (1995), 293–301.

[12] J. O. Carbonara, J. B. Remmel and M. Yang, *A combinatorial rule for the Schur function expansion of the plethysm* $s_{(1^a,b)}[p_k]$, Linear and Multilinear Algebra 39:4 (1995), 341–373.

[13] L. Carini and J. B. Remmel, *Formulas for the expansion of the plethysms* $s_2[s_{(a,b)}]$ *and* $s_2[s_{(n^k)}]$, in Selected papers in honor of A. Garsia (Taormina, 1994), Discrete Math. 193:1-3 (1998), 147–177.

[14] Y. M. Chen, A. M. Garsia and J. Remmel, *Algorithms for plethysm*, in Combinatorics and algebra (Boulder, Colo., 1983), volume 34 of Contemp. Math., 109–153, Amer. Math. Soc., Providence, RI, 1984.

[15] G.-S. Cheon, J. Kim, M. Kim, S. Kitaev and A. Pyatkin, *On k-11-representable graphs*, J. Comb. 10:3 (2019), 491–513.

[16] M. Dukes, S. Kitaev, J. B. and Remmel and E. Steingrímsson, *Enumerating* $(2+2)$*-free posets by indistinguishable elements*, J. Comb. 2:1 (2011), 139–163.

[17] Ö. Eğecioǧlu and J. B. Remmel, *Bijections for Cayley trees, spanning trees, and their q-analogues*, J. Combin. Theory Ser. A 42:1 (1986), 15–30.

[18] Ö. Eğecioğlu and J. B. Remmel, *A combinatorial interpretation of the inverse Kostka matrix*, Linear and Multilinear Algebra 16:1-2 (1990), 59–84.

[19] Ö. Eğecioğlu and J. B. Remmel, *Brick tabloids and the connection matrices between bases of symmetric functions*, Combinatorics and theoretical computer science (Washington, DC, 1989), Discrete Appl. Math. 34:1-3 (1991), 107–120.

[20] Ö. Eğecioğlu and J. B. Remmel, *A bijection for spanning trees of complete multipartite graphs*, in Proceedings of the Twenty-fifth Southeastern International Conference on Combinatorics, Graph Theory and Computing (Boca Raton, FL, 1994), Congr. Numer. 100 (1994), 225–243.

[21] Ö. Eğecioğlu, J. B. Remmel and S. G. Williamson, *A class of graphs which has efficient ranking and unranking algorithms for spanning trees and forests*, Internat. J. Found. Comput. Sci. 15:4 (2004), 619–648.

[22] Ö. Eğecioğlu and J. B. Remmel, *A combinatorial proof of the Giambelli identity for Schur functions*, Adv. in Math. 70:1 (1988), 59–86.

[23] A. M. Garsia and J. B. Remmel, *A combinatorial interpretation of q-derangement and q-Laguerre numbers*, European J. Combin. 1:1 (1980), 47–59.

[24] A. M. Garsia and J. B. Remmel, *Shuffles of permutations and the Kronecker product*, Graphs Combin. 1:3 (1985), 217–263.

[25] A. M. Garsia and J. B. Remmel, *A novel form of q-Lagrange inversion*, Houston J. Math. 12:4 (1986), 503–523.

[26] A. M. Garsia and J. B. Remmel, *Q-counting rook configurations and a formula of Frobenius*, J. Combin. Theory Ser. A 41:2 (1986), 246–275.

[27] A. M. Garsia and J. B. Remmel, *Plethystic formulas and positivity for* q,t*-Kostka coefficients*, in Mathematical essays in honor of Gian-Carlo Rota (Cambridge, MA, 1996), Progr. Math. 161 (1998), 245–262.

[28] J. Haglund, N. Loehr and J. B. Remmel, *Statistics on wreath products, perfect matchings, and signed words*, European J. Combin. 26:6 (2005), 835–868.

[29] J. Haglund, N. Loehr and J. B. Remmel, *Rook theory for perfect matchings*, Special issue in honor of Dominique Foata's 65th birthday (Philadelphia, PA, 2000), Adv. in Appl. Math. 27:2-3 (2001), 438–481.

[30] J. Haglund, N. Loehr and J. B. Remmel, *Cycles and perfect matchings*, Discrete Math. 274:1-3 (2004), 93–108.

[31] J. Haglund, J. B. Remmel and M. Yoo, *p-rook numbers and cycle counting in* $C_p \wr S_n$, in Lattice path combinatorics and applications, Dev. Math. 58, 250–282, Springer, Cham, 2019.

[32] M. Hyatt and J. B. Remmel, *The classification of 231-avoiding permutations by descents and maximum drop*, J. Comb. 6:4 (2015), 509–534.

[33] M. Jones, A. Pyatkin, S. Kitaev and J. B. Remmel, *Representing graphs via pattern avoiding words*, Electron. J. Combin. 22:2 (2015), Article 2.53.

[34] M. E. Jones and J. B. Remmel, *Applying a reciprocity method to count permutations avoiding two families of consecutive patterns*, Pure Math. Appl. (PU.M.A.) 24(2) (2013), 151–178.

[35] M. E. Jones and J. B. Remmel, *A reciprocity method for computing generating functions over the set of permutations with no consecutive occurrence of a permutation pattern*, Discrete Math. 313:23 (2013), 2712–2729.

[36] S. Kitaev, *Patterns in permutations and words*, Monographs in Theoretical Computer Science, An EATCS Series, Springer, Heidelberg, 2011. With a foreword by Jeffrey B. Remmel.

[37] S. Kitaev, J. Liese, J. B. Remmel and B. Sagan, *Rationality, irrationality, and Wilf equivalence in generalized factor order*, in 21st International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2009), Discrete Math. Theor. Comput. Sci. Proc., AK, pages 515–526, Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2009.

[38] S. Kitaev and J. B. Remmel, *Classifying descents according to equivalence mod k*, Electron. J. Combin. 13:1 (2006), Article #R64.

[39] S. Kitaev and J. B. Remmel, *Place-difference-value patterns: a generalization of generalized permutation and word patterns*, Integers 10 (2010) A11, 129–154.

[40] S. Kitaev and J. B. Remmel, *Enumerating $(\mathbf{2}+\mathbf{2})$-free posets by the number of minimal elements and other statistics*, Discrete Appl. Math. 159:17 (2011), 2098–2108.

[41] S. Kitaev and J. B. Remmel, *Quadrant marked mesh patterns*, J. Integer Seq. 15:4 (2012), Article 12.4.7.

[42] S. Kitaev and J. B. Remmel, *Quadrant marked mesh patterns in alternating permutations*, Sém. Lothar. Combin. 68 (2012), Article B68a.

[43] S. Kitaev and J. B. Remmel, *Quadrant marked mesh patterns in alternating permutations II*, J. Comb. 4:1 (2013), 31–65.

[44] S. Kitaev and J. B. Remmel, *(a, b)-rectangle patterns in permutations and words*, Discrete Appl. Math. 186 (2015), 128–146.

[45] S. Kitaev and J. B. Remmel, *A note on p-ascent sequences*, J. Comb. 8:3 (2017), 487–506.

[46] S. Kitaev, J. B. Remmel and M. Tiefenbruck, *Quadrant marked mesh patterns in 132-avoiding permutations*, Pure Math. Appl. (PU.M.A.) 23:3 (2012), 219–256.

[47] S. Kitaev, J. B. Remmel and M. Tiefenbruck, *Quadrant marked mesh patterns in 132-avoiding permutations II*, Integers 15 (2015), Article A16.

[48] S. Kitaev, J. B. Remmel and M. Tiefenbruck, *Quadrant marked mesh patterns in 132-avoiding permutations III*, Integers 15 (2015), Article A39.

[49] T. M. Langley, J. Liese and J. B. Remmel, *Generating functions for Wilf equivalence under generalized factor order*, J. Integer Seq. 14:4 (2011) Article 11.4.2.

[50] T. M. Langley and J. B. Remmel, *The plethysm $s_\lambda[s_\mu]$ at hook and near-hook shapes*, Electron. J. Combin. 11:1 (2004), Artcile #R11.

[51] J. Liese and J. B. Remmel, *Generating functions for permutations avoiding a consecutive pattern*, Ann. Comb. 14:1 (2010) 123–141.

[52] J. LoBue and J. B. Remmel, *A Murnaghan-Nakayama rule for generalized Demazure atoms*, in 25th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2013), Discrete Math. Theor. Comput. Sci. Proc., AS, pages 969–980, Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2013.

[53] N. A. Loehr and J. B. Remmel, *Rook-by-rook rook theory: bijective proofs of rook and hit equivalences*, Adv. in Appl. Math. 42:4 (2009), 483–503.

[54] N. A. Loehr and J. B. Remmel, *A computational and combinatorial exposé of plethystic calculus*, J. Algebraic Combin. 33:2 (2011), 163–198.

[55] A. Mendes and J. B. Remmel, *Permutations and words counted by consecutive patterns*, Adv. in Appl. Math. 37:4 (2006), 443–480.

[56] A. Mendes and J. B. Remmel, *Counting with symmetric functions*, volume 43 of Developments in Mathematics, Springer, Cham, 2015.

[57] A. Mendes, J. B. Remmel and J. Wagner, *A λ-ring Frobenius characteristic for $G \wr S_n$*, Electron. J. Combin. 11:1 (2004), Article #R56.

[58] B. K. Miceli and J. B. Remmel, *Augmented rook boards and general product formulas*, Electron. J. Combin. 15:1 (2008), Article #R85.

[59] R. Pan, D. Qiu and J. B. Remmel, *Counting consecutive pattern matches in $S_n(132)$ and $S_n(123)$*, Adv. in Appl. Math. 105 (2019), 105–130.

[60] D. Qiu and J. B. Remmel, *Quadrant marked mesh patterns in 123-avoiding permutations*, Discrete Math. Theor. Comput. Sci. 19:2 (2017), Article 12.

[61] D. Qiu and J. B. Remmel, *Schur function expansions and the rational shuffle conjecture*, Sém. Lothar. Combin. 78B (2017), Article 83.

[62] D. Qiu and J. B. Remmel, *Classical pattern distributions in $S_n(132)$ and $S_n(123)$*, Discrete Math. Theor. Comput. Sci. 21:2 (2019), Paper No. 4.

[63] D.J. Qiu and J. B. Remmel, *Schur function expansions and the rational shuffle theorem*, J. Combin. Theory Ser. A, 175 (2020) 105272.

[64] A. Ram, J. B. Remmel and T. Whitehead, *Combinatorics of the q-basis of symmetric functions*, J. Combin. Theory Ser. A 76:2 (1996), 231–271.

[65] J. B. Remmel, *Bijective proofs of formulae for the number of standard Young tableaux*, Linear and Multilinear Algebra 11:1 (1982), 45–100.

[66] J. B. Remmel, *Bijective proofs of some classical partition identities*, J. Combin. Theory Ser. A 33:3 (1982), 273–286.

[67] J. B. Remmel, *The combinatorics of (k,l)-hook Schur functions*, in Combinatorics and algebra (Boulder, Colo., 1983), volume 34 of Contemp. Math. 253–287, Amer. Math. Soc., Providence, RI, 1984.

[68] J. B. Remmel, *Permutation statistics and (k,l)-hook Schur functions*, Discrete Math. 67:3 (1987), 271–298.

[69] J. B. Remmel, *A formula for the Kronecker products of Schur functions of hook shapes*, J. Algebra 120:1 (1989), 100–118.

[70] J. B. Remmel, *A bijective proof of a factorization theorem for (k,l)-hook Schur functions*, Linear and Multilinear Algebra 28:3 (1990), 119–154.

[71] J. B. Remmel, *Combinatorial algorithms for the expansion of various products of Schur functions*, Acta Appl. Math. 21:1-2 (1990), 105–135.

[72] J. B. Remmel, *Formulas for the expansion of the Kronecker products $S_{(m,n)} \otimes S_{(1^{p-r},r)}$ and $S_{(1^k2^l)} \otimes S_{(1^{p-r},r)}$*, Discrete Math. 99:1-3 (1992), 265–287.

[73] J. B. Remmel, *Consecutive up-down patterns in up-down permutations*, Electron. J. Combin. 21:3 (2014) Paper 3.2.

[74] J. B. Remmel and J. LoBue Tiefenbruck, *Q-analogues of convolutions of Fibonacci numbers*, Australas. J. Combin. 64 (2016), 166–193.

[75] J. B. Remmel and M. Shimozono, *A simple proof of the Littlewood-Richardson rule and applications*, volume 193, pages 257–266, 1998, selected papers in honor of A. Garsia (Taormina, 1994).

[76] J. B. Remmel and Michelle L. Wachs, *Rook theory, generalized Stirling numbers and (p,q)-analogues*, Electron. J. Combin. 11:1 (2004), Article #R84.

[77] J. B. Remmel and T. Whitehead, *On the Kronecker product of Schur functions of two row shapes*, Bull. Belg. Math. Soc. Simon Stevin 1:5 (1994), 649–683.

[78] J. B. Remmel and T. Whitehead, *Transition matrices and Kronecker product expansions of symmetric functions*, Linear and Multilinear Algebra 40:4 (1996), 337–352.

[79] J. B. Remmel and R. Whitney, *A bijective proof of the hook formula for the number of column strict tableaux with bounded entries*, European J. Combin. 4:1 (1983), 45–63.

[80] J. B. Remmel and R. Whitney, *A bijective proof of the generating function for the number of reverse plane partitions via lattice paths*, Linear and Multilinear Algebra 16:1-4 (1984), 75–91.

[81] J. B. Remmel and R. Whitney, *Multiplying Schur functions*, J. Algorithms 5:4 (1984), 471–487.

[82] J. B. Remmel and A. T. Wilson, *Block patterns in Stirling permutations*, J. Comb. 6:1-2 (2015), 179–204.

[83] J. B. Remmel and M. Yang, *Special rim hook tabloids and some new multiplicity-free S-series*, SIAM J. Discrete Math. 4:2 (1991), 253–274.

[84] M. Yang and J. B. Remmel, *Hook-Schur functions analogues of Littlewood's identities and their bijective proofs*, Europ. J. Comb. 19:2 (1998), 257–272.