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Interview with Victor Reiner

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Victor Reiner completed his undergraduate studies at Princeton University in 1986. He obtained a Ph.D. from the Massachusetts Institute of Technology in 1990, under the supervision of Richard Stanley. After that, he was postdoc at the University of Minnesota and has remained there ever since, as an Assistant, Associate, and then Full Professor in 2001. Professor Reiner has given numerous invited talks in conferences and seminars. Since 2012 he is a Fellow of the American Mathematical Society (AMS). Professor Reiner has served as a member of the editorial board in numerous journals, including Journal of the AMS (2004–2009), ORDER(1998-2007), Journal of Algebraic Combinatorics (2000–2017), Journal of Combinatorial Theory Series A (2015–2020), Algebra and Number Theory (2007–), Algebraic Combinatorics (2018-), Combinatorial Theory (2021-).

Mansour: Professor Reiner, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Reiner: To me, it is the mathematics of the discrete. But it is *so* hard to separate from other areas. Joszef Balogh and I face this problem all the time with submissions to the `arXiv` category `math.CO`, which we co-moderate. It is often fruitless to draw a distinction between combinatorics and other subjects.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Reiner: It is great! I am always looking for excuses to get my undergraduate and graduate students to pay attention in *all* of their courses, not just combinatorics.

Mansour: What have been some of the main goals of your research?

Reiner: I have spent a lot of time trying to understand which parts of "classical" combinatorics pertain to the symmetric group, and are special cases of results about Weyl groups, Coxeter groups, complex reflection groups, or even more general reflection groups connected with invariant theory. This is a thread with many inspiring 20th century results, by people like Björner, Coxeter, Garsia, Lusztig, Orlik, Solomon, Shephard, Stanley, Steinberg, Terao, and Wachs.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Reiner: My family was not enthusiastic about my choice to go into math, but they forgave me. My parents were Polish Jews who had survived World War II in hiding and met after both had immigrated to the US. My father was an electrical engineer, who claimed that he did not really enjoy math, per se. My mother was a physician, as was her father, and there was an expectation within the family that everyone would grow up to be a physician. I grew

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up thinking this way too but slowly started to realize that I liked math better than the sciences. My parents eventually reconciled to this, but my grandmother was more adamant: when I started graduate school, she said she hoped that I would fail, and go back to a medical school.

Mansour: Were there specific problems that made you first interested in combinatorics?

Reiner: I was extremely lucky to have taken the MIT class Math 18.318 Topics in Combinatorics five times, twice from Mark Haiman and three times from Richard Stanley. All five classes were great, and I still cherish my notebooks from them! One of them by Stanley, on combinatorial commutative algebra, had a unit on the invariant theory of finite groups and mentioned some problems that enticed me. After I worked on some of them and expressed interest in this topic, Stanley suggested that I talk to Ira Gessel, who told me about another problem that got me hooked. Thank you, Ira!

Mansour: What was the reason you chose the Massachusetts Institute of Technology (MIT) for your Ph.D. and your advisor Richard Stanley?

Reiner: I think MIT was the best place that I was admitted. I had done an undergraduate thesis related to the probabilistic method in combinatorics, with Michael Steele, and enjoyed it. When I got to MIT, one of my officemates was Maciej Zworski, a great analyst now at UC Berkeley. When I told him that I liked combinatorics, a topic that he knew something about, he unequivocally said Stanley was the best advisor choice among the three combinatorialists there at the time (Kleitman, Rota, Stanley). Thank you, Maciej!

Mansour: What was the problem you worked on in your thesis?

Reiner: This was the problem mentioned above that came from Ira Gessel. He and Adriano Garsia had proven a permutation statistics result using the theory of multipartite P-partitions[1], closely connected with the diagonally symmetric functions studied by MacMahon, Solomon, and others. So, Ira suggested that I pursue an approach to understanding this via invariant theory, as pioneered in an important paper by Garsia and Stanton[2]. Ira was right – it worked and led me to lots of other related things.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Reiner: General theoretical questions. I am a pretty weak problem-solver!

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Reiner: Yes, if I (or the computer) have done enough examples. Or if it is just too beautiful *not* to be true, once it has been properly formulated.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Reiner: Well, I am biased toward the combinatorics that I know better, what one might call the"Rota-Stanley School". So I can not fully appreciate things like Szemeredi's Regularity Lemma[3] or Keevash's result on the existence of designs[4]. I would list (1) Haiman's[5] "$(n+1)^{n-1}$ and $n!$ Theorems", (2) Adiprasito, Huh, and Katz's[6] proof of the Rota-Heron Welsh and Mason Conjectures, and (3) Elias and Williamson's[7] proof of the Kazhdan-Lusztig Conjecture.

Mansour: What are the top three open questions in your list?

Reiner: (1) Combinatorial interpretation of the Schubert calculus structure constants in the type A flag manifold[8]. (2) The Kronecker problem[9]: combinatorial interpretation of structure constants for tensor products of symmetric group irreducible characters. (3)

[1] I. Gessel, *A historical survey of P-partitions*, The Mathematical Legacy of Richard P. Stanley, Amer. Math. Soc., Providence, RI, 2016, pp. 169–188.

[2] A. M. Garsia and D. Stanton, *Group actions on Stanley-Reisner rings and invariants of permutation groups*, Adv. Math. 51:2 (1984), 107–201.

[3] T. Tao, *Szemerédi's regularity lemma revisited*, Contrib. Discrete Math. 1:1 (2006), 8–28

[4] See `http://people.maths.ox.ac.uk/keevash/papers/designsI.pdf`.

[5] M. Haiman, *Combinatorics, symmetric functions and Hilbert schemes*, Current Developments in Mathematics, 2001, 30–111.

[6] K. Adiprasito, J. Huh, and E. Katz, *Hodge theory for combinatorial geometries*, Ann. of Math. 188 (2018), 381–452.

[7] B. Elias and G. Williamson, *The Hodge theory of Soergel bimodules*, Ann. of Math. 180 (2014), 1–48.

[8] P. Pragacz, *Multiplying Schubert classes.* Topics in cohomological studies of algebraic varieties, 163–174, Trends Math., Birkhäuser, Basel, 2005.

[9] I. Pak and G. Panova, *On the complexity of computing Kronecker coefficients*, Comput. Complexity 26:1 (2017), 1–36.

[10] S. R. Gal, *Real root conjecture fails for five- and higher-dimensional spheres*, Discrete Comput. Geom. 34 (2005), 269–284.

The Charney-Davis-Gal[10] conjecture on the nonnegativity of γ-vectors for flag simplicial spheres.

Mansour: Are there aspects of a career in mathematics, or particularly in combinatorics, that surprised you?

Reiner: A few, it was a wonderful surprise to find so many collaborative, helpful, and extremely humble combinatorial colleagues, all over the world. I love the fact that, as mathematicians, we get to travel and meet people from nearly everywhere. It was also unexpected, gratifying, and comforting to realize that we would form relationships with colleagues that span decades.

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

Reiner: Continuing my work is not so important. I just hope we see more cool new ideas, like some that have been springing up recently.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Reiner: Not really. It is more about what people make of the topic, and how much beautiful structure they reveal within it.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Reiner: Be ready for a lot of ups and downs. And not to be too trite, but "the race is not always to the swift" – I am thankful for that! I just feel lucky that the enterprise of math research is currently a large enough table to have a seat for all of us.

Mansour: Would you tell us about your interests besides mathematics?

Reiner: I am pretty boring – I like to run, watch movies, do crossword puzzles, travel, and visit museums.

Mansour: You have recently been involved in creating new combinatorics journals, such as *Algebraic Combinatorics*, and *Combinatorial Theory* whose editorial boards came from mass-resignations of the boards of journals owned by large commercial publishers. Why was it important to do this?

Reiner: Thanks for asking, and I applaud you and your co-editors for your vision in using the *diamond open access* model (no fees for authors or readers) in creating this new journal, *Enumerative Combinatorics and I Applications*. I think it is important for mathematics to pry away its best journals from the control of commercial publishers like Springer-Nature and Elsevier because they put our articles behind paywalls and charge our libraries exorbitant, non-transparent, bundled subscription rates. Sadly, in combinatorics, up until recently all of the oldest and best-known journals were owned by these two publishers. Founding new journals like ECA is part of the solution, and eliminating the need for people to send papers to the commercially-published journals is another part of it.

Mansour: You have mentored more than 100 undergraduate students in Research Experiences for Undergrads (REU) program. Would you tell us about your experience in this program? How important is it for an undergraduate student to have research experience under an experienced researcher's supervision for their future research career? You probably follow your students' career after their undergraduate research experience and see that some of them build a successful research career, but some do not. What is the main factor that distinguishes these two groups? Ambition, hard work, talent, intelligence ...?

Reiner: For good or bad, it has become very hard to gain entrance to a top Ph.D. program in the US without something similar to an REU research experience. And being honest, we at Minnesota have been pretty lucky that our REU program has been around for many years so that it now gets some of the best students. They are not only talented, but very hard-working, and I think the latter is the more important factor in their success.

Mansour: What advice would you give to faculty members who mentor younger researchers in their research careers? How would you describe a great teacher, mentor?

Reiner: It is tricky knowing when your own instincts as a mentor are right, and you need to push the mentee harder to follow your suggestion(s), versus when to get out of their way because their own ideas are better! It is even trickier to figure out when a mentee who started out in the first situation has grown to the point where they are more often in the second situation.

Mansour: You have also guided numerous talented young undergraduate students to publish their first papers from their REU project. Have you seen some results that you consider outstanding?

Reiner: One of my favorites was the work of David B. Rush and Danny Shi[11] from our Summer 2011 REU. They insightfully used known combinatorics of minuscule posets to prove our conjectures in what Jim Propp calls *dynamical algebraic combinatorics*. These conjectures were about an enumerative phenomenon that Dennis Stanton, Dennis White, and I call a *cyclic sieving phenomenon*[12], and dealt with a cyclic action now called *rowmotion* that had been considered by Peter Cameron and Dmitry Fon-der-Flaass[13]. Rush and other collaborators later proved even more beautiful results about this picture, which ended being very influential for the people working in this area.

Mansour: One of your research interests is on *Permutation statistics*. How would explain the importance of such statistics in studying combinatorial structures?

Reiner: Some permutation statistics arise naturally in writing down *Hilbert series* for invariant rings with respect to different gradings. That is, the generating functions for those statistics track the dimensions of the graded components of these rings. I am thinking here of things like the number of descents, the major index, the number of inversions. But then others also arise in Hilbert series of other rings, like the number of cycles. All of these permutation statistics generalize in some way to statistics on other reflection groups besides the symmetric group.

Mansour: Several families of algebras are related to combinatorics. How do Hopf algebras connect to combinatorics in general and enumeration in particular?

Reiner: People have found Hopf algebras that re-interpret many of our favorite recurring families of combinatorial numbers or objects, like partitions, compositions, permutations, tableaux, graphs, simplicial complexes, posets, trees, Catalan numbers, Baxter permutations, alternating sign matrices, and more. When you have such a Hopf algebra, it starts making you think about a new set of questions pertaining to the objects: What are the most natural ways of taking them apart or putting them together? Who are the indecomposable or primitive objects? What are the natural maps between the objects?

Mansour: You mentioned the *cyclic sieving phenomenon* that you have worked on with Stanton and White. How did that work arise? What aspects of it do you find most important?

Reiner: It arose in a strange way. The “Denises” (Stanton and White)[14] and I ran across an interesting enumerative coincidence that Frédéric Chapoton had posted on his website[15], counting cyclically symmetric rooted trees in terms of q-binomial coefficients reduced modulo $q^n - 1$. In unpacking Chapoton’s empirical observation, we started running across similar-sounding results all over the place. We later realized that they could be reformulated to generalize John Stembridge’s famous “$q = -1$ phenomenon”, and then started finding it happening in even more places.

This reformulation also highlights for me some of the importance: beautiful q-counts for combinatorial objects can be even more beautiful because they also count the same objects with cyclic symmetry when you set q to a root-of-unity. And you do not have to memorize more formulas, just the single q-formula!

It was also very gratifying for me when I realized that many of these cyclic sieving phenomena were explained by results from invariant theory, including a beautiful theorem of the late Tonny Springer[16].

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Reiner: Since my enumerative skills are not as strong as some, I mainly use enumerative answers and data as a ”fingerprint” to sniff out situations where there is a more interesting structure, a philosophy espoused by Sara Billey and Bridget Tenner. That is, if the counts or the generating functions for the objects look

[11] D. B. Rush and X. Shi, *On orbits of order ideals of minuscule posets*, J. Algebraic Combin. 37:3 (2013), 545–569.
[12] V. Reiner, D. Stanton, and D. White, *What is ... Cyclic Sieving?*, Notices Amer. Math. Soc. 61:2 (2014), 169–171.
[13] P. J. Cameron and D. G. Fon-der-Flaass, *Orbits of Antichains Revisited*, European J. Combin. 16:6, (1995), 545–554.
[14] V. Reiner, D. Santon, and D. White, *The cyclic sieving phenomenon*, J. Combin. Theory, Ser. A 108:1 (2004), 17–50.
[15] See `https://irma.math.unistra.fr/~chapoton/`.
[16] T. A. Springer, *Regular elements of finite reflection groups*, Invent. Math. 25 (1974), 159–198.

beautifully simple, then there is probably a lot more interesting structure (poset-theoretic, algebraic, topological) lurking.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Reiner: Such a moment came in formulating the main conjecture in the paper "Parking spaces" with Drew Armstrong and Brendon Rhoades[17]. This conjecture would provide a satisfactory explanation for a mystery that has plagued us for over 20 years: why do the Catalan numbers from a reflection group W, which are given by a simple product formula, count the W-noncrossing partitions or the W-clusters? At some moment it hit me that this might be explained by a "deformation" method from geometry that Bram Broer had taught me earlier, in a slightly different invariant theory context. I still have hopes that this deformation method might be the key to similar conjectures, e.g., some by John Shareshian and Michelle Wachs related to the famous Stanley-Stembridge[18] Conjecture on (3+1)-free posets.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Reiner: The Parking Space Conjecture that I just mentioned has seen subsequent work of Rhoades and of Theo Douvropoulos, connecting it to even more mysteries, making it more tantalizing. Some work that I did with Joel Lewis and Dennis Stanton[19] found an interesting analog for finite general linear groups. But this problem eludes me.

Mansour: Professor Victor Reiner, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

[17] D. Armstrong, V. Reiner, and B. Rhoades, *Parking spaces*, Adv. Math. 69:10 (2015), 647–706.

[18] R. P. Stanley, and J. R. Stembridge, *On immanants of Jacobi-Trudi matrices and permutations with restricted position*, J. Combin. Theory, Ser. A 62:2 (1993), 261–279.

[19] J. B. Lewis, V. Reiner, and D. Stanton, *Invariants of $GL_n(F_q)$ mod Frobenius powers*, Proc. Roy. Soc. Edinburgh Sect. A 147:4 (2017), 831–873.