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Interview with Carla D. Savage

Toufik Mansour

Photo by Griff Bilbro

Carla Savage completed her bachelor's degree studies at Case Western Reserve University in 1973 and her master's studies at the University of Illinois, Urbana-Champaign in 1975. She obtained a Ph.D. from the University of Illinois at Urbana–Champaign in 1977, under the supervision of David E. Muller. She is a Professor in the Department of Computer Science at North Carolina State University. Since 2012, she is a Fellow of the AMS. In 2019, she became a SIAM Fellow "for outstanding research in algorithms of discrete mathematics and computer science applications, alongside exemplary service to mathematics." Her research interests include Combinatorics; enumeration and structure in combinatorial families; theory of partitions; linear Diophantine enumeration; lattice point enumeration; permutation statistics; the combinatorics, geometry, and number theory of lecture hall partitions. Professor Savage has given numerous invited talks in conferences and seminars. From 2013 up to the end of January this year, she was the secretary of the American Mathematical Society.

Mansour: Professor Savage, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Savage: To me, it is the study of enumeration and structure in discrete families.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Savage: I have been most aware of relations between combinatorics and theoretical computer science. Since the early seventies, the areas have grown up together, each challenging the other with intriguing problems. Some of the most recognized leaders in combinatorics have had a strong interest in computation.

Mansour: What have been some of the main goals of your research?

Savage: When I turned my attention to combinatorics, I learned of so many interesting open questions. I was always thinking, "What can I compute that would give insight into the solution of a problem, that would help a mathematician solve the problem?"

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Savage: I went to public school in Baltimore County, Maryland. We could take advanced courses in many areas, for instance, mathematics, physics, and french. Amazing teachers. Small classes with students who were motivated, competitive, fun.

Mansour: Were there specific problems that made you first interested in combinatorics?

Savage: I attended the 1988 SIAM Conference on Discrete Mathematics in San Francisco for the chance to hear Donald Knuth speak. But I also went to Herb Wilf's talk there on Gray codes and was intrigued by some of his

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questions. For example, is it possible to list all partitions of an integer n, each exactly once, so that, in moving from one partition to the next, one part of the partition increases by 1 and one part decreases by 1? In working that out, I fell in love with integer partitions. That changed everything.

Mansour: What was the reason you chose the University of Illinois at Urbana–Champaign for your Ph.D. and your advisor David E. Muller?

Savage: My husband and I were looking at schools with strong graduate programs in mathematics and physics. UIUC suited us both well. I had interests in electrical engineering and computer science as well as mathematics, things like coding theory, switching theory, fault-tolerant computing, automata theory, formal languages, algorithms, graph theory, and complexity theory. David Muller was a mathematician who thought very abstractly about problems in computer science, so I was happy that he was willing to take me on. He is the "Muller" of Reed-Muller codes.

Mansour: What was the problem you worked on in your thesis?

Savage: I worked on the design of parallel algorithms for graph problems. By the mid-seventies, parallel computers were already being imagined, designed, and built: the Illiac IV, STARAN, CRAY. On the horizon were machines like the Intel Hypercube, the Connection Machine. And there were a variety of models of parallel computation. One of those models, the PRAM (parallel random-access machine) assumed that an unlimited, but finite, number of processors could work in sync, with random access to a common memory, just about the best you hope for. What speedup could be achieved with such a model? It turns out that many problems that can be solved in polynomial time $O(n^k)$ on a sequential computer can be solved in poly-logarithmic $O((\log n)^k)$ time on a PRAM with a polynomial number of processors.

This work was coming after a decade of great strides in the design of efficient sequential graph algorithms by John Hopcroft, Robert Tarjan, and others. However, one of their most powerful tools - depth-first search - seems to be inherently sequential. So other methods had to be developed for the PRAM.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Savage: Usually, a specific problem. It is fun to learn something new if it might help to solve your problem.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Savage: All the time, but I am often wrong.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Savage: Here are some things in my realm of interest:

(1) Middle levels problem. Consider the lattice of subsets of a $2k+1$-element set ordered by inclusion. Is there a Hamiltonian cycle in the bipartite graph formed by the k- and $k+1$-element subsets? So easy to state, but many people spent a lot of time on this problem without solving it. Finally in a 2016 paper published in *Proc. London Math. Soc.*, Torsten Mütze[1] showed constructively that it is always possible. In the years since, progress has been made on several related questions. It has led to a consolidation of ad hoc techniques for Gray codes and simplifications of previous algorithms.

(2) Four color theorem. More than thirty years ago, I know, but this was happening while I was a graduate student at Illinois, home of Kenneth Appel and Wolfgang Haken[2], and it was pretty exciting. It influenced so much of the work of the past forty years, on graph minors and the structure of graphs and, correspondingly, on the design of efficient algorithms for special classes of graphs.

(3) Barvinok's algorithm[3] for lattice point enumeration. In 1994, Alexander Barvinok gave the first polynomial-time algorithm for counting the number of lattice points in a convex polyhedron in any fixed dimension d. The algorithm is polynomial in the dimension d and the size of the problem. This led to the development of software for efficient lattice point enumeration such as LattE and its successors.

[1] T. Mütze, *Proof of the middle levels conjecture*, Proc. Lond. Math. Soc. (3) 112 (2016), no. 4, 677–713.
[2] K. Appel and W. Haken, *A proof of the four color theorem*, Discrete Math. 16 (1976), no. 2, 179–180.
[3] A. Barvinok, *Polynomial time algorithm for counting integral points in polyhedra when the dimension is fixed*, Math. Oper. Res. 19 (1994), 769–779.

Mansour: What are the top three open questions in your list?

Savage: (1) Does $L(m, n)$ have a symmetric chain decomposition? $L(m, n)$ is the lattice of integer partitions whose Ferrers diagrams fit in an $m \times n$ box, ordered by diagram inclusion. In 1980, Richard Stanley[4] proved that $L(m, n)$ is rank symmetric and unimodal and conjectured that it had the stronger property of possessing a decomposition into symmetric chains. The question of a symmetric chain decomposition when $m \geq 5$ remains open at this time, although Kathy O'Hara[5] gave an elegant combinatorial proof of unimodality of $L(m, n)$ in 1990.

(2) Is the Durfee polynomial real-rooted? The Durfee polynomial is defined by $D_n(x) = \sum_{i\geq 0} p(n, d)x^d$ where $p(n, d)$ is the number of partitions of n with Durfee square of size d. With Rod Canfield and Sylvie Corteel[6] in 1998 we showed that, as n tends to infinity, the sequence of coefficients of $D_n(x)$ is asymptotically normal, unimodal, and log-concave. However, empirical evidence led us to conjecture that $D_n(x)$ has only real roots for all positive n, a stronger property which would also imply that the average size and most likely size of the Durfee square of a partition of n differ by at most 1.

(3) Do simple symmetric n-Venn diagrams exist for all prime n? With Jerry Griggs and Chip Killian[7], we showed in 2004 that rotationally symmetric Venn diagrams for n sets (like the familiar one for three sets) exist for all prime n. But there may be points where 3 or more of the n curves intersect. Can the same result be achieved with simple Venn diagrams in which at most two curves intersect at a point? This was known to be possible for $n = 1, 2, 3, 5, 7$ and more recently, thanks to Khalegh Mamakani and Frank Ruskey[8], for $n = 11, 13$.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Savage: For many years the AMS Steele Prize for Seminal Contribution to Research was given in five "core" areas of mathematics, with the prize being awarded in one area each year on a five-year cycle. A few years ago, the five core areas were redefined to better cover research mathematics: Analysis/Probability, Algebra/Number Theory, Applied Mathematics, Geometry/Topology, and Discrete Mathematics/Logic. In addition, a sixth category "Open" was included, resulting in a six-year cycle for the prize. I think this is a recognition that there are core areas, but that important research these days may cross boundaries.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Savage: I am probably seen as an applied mathematician since I have spent my whole career in a computer science department. But even within computer science, there are more theoretical and more applied areas. Both had important roles to play in the progress of the last fifty years, thanks to researchers with the vision to put them together.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Savage: Learn what mathematics is good for. I do not think my generation did that very well.

Mansour: Would you tell us about your interests besides mathematics?

Savage: Family, film, books, music, nature, simple pleasures.

Mansour: You have been the Secretary of the American Mathematical Society for eight years. Would you tell us about your experiences? How do you manage to balance your time among research, teaching, and your professional service?

Savage: My experience with the AMS has left me in awe of the tremendous service mathematicians are willing to do for our field. There are hundreds of volunteers appointed each year

[4]R. P. Stanley, *Weyl groups, the hard Lefschetz theorem, and the Sperner property*, SIAM J. Algebraic and Discrete Methods 1 (1980), 168–184.

[5]K. O'Hara, *Unimodality of Gaussian coefficients: a constructive proof*, J. Comb. Theory, Ser. A 53 (1990), 29–52.

[6]E.R. Canfield, S. Corteel, and C.D. Savage, *Durfee polynomials*, Elec. J. Comb. 5 (1998), R32.

[7]J. Griggs, C. E. Killian, and C. D. Savage, *Venn diagrams and symmetric chain decompositions in the boolean lattice*, Elec. J. Comb. 11:1 (2004), #R2.

[8]K. Mamakani and F. Ruskey, *New roses: simple symmetric Venn diagrams with* 11 *and* 13 *curves*, Discrete Comput. Geom. 52:1 (2014), 71–87.

to editorial and other AMS committees. I think it is a challenge for all of us to balance our time between research, teaching, and professional service. As AMS secretary, I was involved in some way with most aspects of the society: publications, conferences, prizes, committees, elections. I was in a unique position to see how the pieces fit together and to work behind the scenes to help make things happen. I am grateful to have had that opportunity.

Mansour: You have written an excellent survey paper on Gray codes titled as *A survey of combinatorial Gray codes*[9]. Would you tell us about Gray codes and their combinatorial aspects? There is also a connection between Gray codes and music theory. Would you explore more about this connection?

Savage: A Gray code for a combinatorial family is a listing of the objects in the family, each exactly once, in such a way that successive objects differ only in a prescribed (usually small) way. For example, the binary reflected Gray code lists all n-bit strings so that successive strings differ in only one-bit position. A Gray code for permutations might list all permutations of $\{1, \dots, n\}$ so that successive permutations differ only by a transposition; there are many other conditions on successive elements that one might prescribe. A permutation Gray code can be interpreted as a score for ringing n church bells in $n!$ rounds, where each permutation specifies the order in which to ring the n bells in that round.

A Gray code can be viewed as a Hamiltonian path in the graph whose vertices are the objects in the combinatorial family and where objects are adjacent if they differ by the prescribed change.

Gray code schemes typically involve decomposing a combinatorial structure into smaller substructures which are then listed recursively. How to combine the sublists to enforce the Gray code restriction at the boundaries is the hard part. Frank Ruskey and his former students Joe Sawada and Aaron Williams have been responsible for (and continue to produce) some of the most delicately clever Gray code constructions. Donald Knuth included several results and open questions about Gray codes in Volume 4A of *The Art of Computer Programming*.

Mansour: In a very recent short article, published at Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson[10], while elaborating on the ethical aspects of the question "Who Owns the Theorem?" concluded that "Mathematical truths exist, and mathematicians only discover them." On the other hand, some people claim that "mathematical truths are invented," and some others claim that it is both invented and discovered. What do you think about this old discussion?

Savage: I found Nathanson's article quite interesting, especially the ethical issues raised. It reminds me of questions about whether algorithms can be patented - in most cases, they cannot be, they are considered abstract ideas. But the software implementation of the algorithm might be.

I think mathematical truths are discovered. But perhaps, sometimes the questions that lead to their discovery are invented.

Mansour: *Lecture Hall partitions* has recently become an active area of research and you have some interesting works in this direction. Why are they important? Do they have some surprising connections with other combinatorial objects?

Savage: Lecture hall partitions L_n, introduced by Mireille Bousquet-Mèlou and Kimmo Eriksson[11] in 1997, are integer sequences $\lambda = \lambda_1, \dots, \lambda_n$ satisfying

$$0 \leq \frac{\lambda_1}{1} \leq \frac{\lambda_2}{2} \leq \dots \leq \frac{\lambda_n}{n}.$$

Bousquet-Mèlou and Eriksson showed that

$$\sum_{\lambda \in L_n} q^{\sum_{i=1}^n \lambda_i} = \frac{1}{\prod_{j=1}^n (1 - q^{2j-1})},$$

a result known as the lecture hall theorem. It is surprising that these partitions, defined by the ratio of consecutive parts, would have such a simple generating function. Moreover, the right-hand side is the generating function for partitions into odd parts less than $2n$. In fact, the lecture hall theorem is a new finite version of Euler's theorem that the number of partitions of an integer into distinct parts is the same as the number of its partitions into odd

[9]C. D. Savage, *A survey of combinatorial Gray codes*, SIAM Rev. 39:4 (1997), 605–629.

[10]See `https://www.ems-ph.org/journals/newsletter/pdf/2020-12-118.pdf`.

[11]M. Bousquet-Mélou and K. Eriksson, *Lecture hall partitions*, Ramanujan J. 1:1 (1997), 101–111.

parts. Bousquet-Mèlou and Eriksson[12] also defined s-lecture hall partitions $L_n^{(s)}$ for nonnegative integer sequences s as sequences λ satisfying

$$0 \leq \frac{\lambda_1}{s_1} \leq \frac{\lambda_2}{s_2} \leq \ldots \leq \frac{\lambda_n}{s_n}$$

and looked for sequences s that would yield interesting enumerative results. As one example, they showed that if s is defined, for positive integer ℓ, by $s_n = \ell s_{n-1} - s_{n-2}$ with $s_0 = 0$, $s_1 = 1$, then

$$\sum_{\lambda \in L_n^{(s)}} q^{\sum_{i=1}^n \lambda_i} = \frac{1}{\prod_{j=1}^n (1 - q^{s_{j-1}+s_j})}.$$

Since then, many others have looked at aspects of lecture hall partitions. As I wrote in my 2016 survey paper, *The Mathematics of Lecture Hall Partitions*[13], over the past 25 years lecture hall partitions have emerged as fundamental structures in combinatorics, number theory, algebra, and geometry, leading to new generalizations and interpretations of classical theorems and new results.

Mansour: In joint work with Michael J. Schuster, *Ehrhart series of lecture hall polytopes and Eulerian polynomials for inversion sequences*[14], you introduced s-lecture hall polytopes, s-inversion sequences and studied relevant statistics on both families. What was the main motivation behind their studies?

Savage: We found earlier[15] that the number of lecture hall partitions λ with $\lambda_n/n \leq t$ is $(t+1)^n$. This suggests a connection with permutations since the Eulerian polynomial $E_n(x)$ satisfies

$$\sum_{t \geq 0} (t+1)^n x^t = \frac{\sum_{\pi \in S_n} x^{des(\pi)}}{(1-x)^{n+1}} = \frac{E_n(x)}{(1-x)^{n+1}}.$$

I was first able to explain the connection bijectively in a paper with Katie Bright[16] in 2010. In 2012, with Michael Schuster, we were looking for a generalization for s-lecture hall partitions, but that required a way to generalize permutations. Mike's idea was to use s-inversion sequences $I_n^{(s)}$, defined as

$$I_n^{(s)} = \{e \in Z^n \mid 0 \leq e_i < s_i,\ i = 1, \ldots, n\}.$$

A key ingredient to make things work was to define the ascent statistic over $I_n^{(s)}$ in the right way. Position i is an *ascent* in s-inversion sequence e if $e_i/s_i < e_{i+1}/s_{i+1}$. And position 0 is an ascent if $0 < e_1$. With this definition we were able to show bijectively that if $i_n^{(s)}(t)$ is the number of s-lecture hall partitions with $\lambda_n/s_n \leq t$, then

$$\sum_{t \geq 0} i_n^{(s)}(t) x^t = \frac{\sum_{e \in I_n^{(s)}} x^{asc(e)}}{(1-x)^{n+1}}.$$

We called the numerator of the right-hand side the s-Eulerian polynomial $E_n^{(s)}(x)$ and defined the s-lecture polytope $P_n^{(s)}$ as the set of all *real* points $\lambda \in R^n$ satisfying

$$0 \leq \frac{\lambda_1}{s_1} \leq \frac{\lambda_2}{s_2} \leq \ldots \leq \frac{\lambda_n}{s_n} \leq 1.$$

Then $i_n^{(s)}(t)$ is the Ehrhart polynomial of the s-lecture hall polytope. And $E_n^{(s)}$ is the h^*-polynomial of the s-lecture hall polytope, as well as the ascent polynomial of the s-inversion sequences. Following the bijection allowed us to define and track several other meaningful statistics.

Mansour: In joint work with Mirko Visontai, *The s-Eulerian polynomials have only real roots*[17], you proved a conjecture of Brenti, showing that Eulerian polynomials for all finite Coxeter groups have only real roots. Therein, you also partially settled a conjecture of Dilks, Petersen, and Stembridge on type-B affine Eulerian polynomials. Further, by extending results to q-analogs, you have shown that the MacMahon–Carlitz q-Eulerian polynomial has only real roots whenever q is a positive real number, thus confirming a conjecture of Chow and Gessel. In the end, you provided the arguments of the validity of the results for the case

[12] M. Bousquet-M èlou and K. Eriksson, *Lecture hall partitions. II*, Ramanujan J. 1(2) (1997), 165–185.

[13] C. D. Savage, *The mathematics of lecture hall partitions*, J. Combin. Theory Ser. A 144 (2016), 443–475.

[14] C. D. Savage and M. J. Schuster, *Ehrhart series of lecture hall polytopes and Eulerian polynomials for inversion sequences*, J. Combin. Theory Ser. A 119:4 (2012), 850–870.

[15] S. Corteel, S. Lee, and C. D. Savage, *Enumeration of sequences constrained by the ratio of consecutive parts*, S èm. Lothar. Combin. 54A (2005/07), Article B54Aa.

[16] K. L. Bright and C. D. Savage, *The geometry of lecture hall partitions and quadratic permutation statistics*, In 22nd International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2010), Discrete Math. Theor. Comput. Sci. Proc., AN, pages 569–580. Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2010.

[17] C. D. Savage and M. Visontai, *The s-Eulerian polynomials have only real roots*, Trans. Amer. Math. Soc. 367:2 (2015), 1441–1466.

of the hyperoctahedral group and the wreath product groups, confirming further conjectures of Chow and Gessel, and Chow and Mansour, respectively. Would you tell us about the important ideas behind the proofs?

Savage: Analogous to the way the Eulerian polynomial is viewed as the descent polynomial of permutations, the s-Eulerian polynomials $E_n^{(s)}(x)$ are the ascent polynomials of the s-inversion sequences. There is a bijection between inversion sequences and permutations that sends ascents to descents. Similarly, we showed there is a bijection between $(2, 4, \ldots, 2n)$-inversion sequences and signed permutations that send ascents to descents. So the s-Eulerian polynomials for $s = (1, 2, \ldots, n)$ and $s = (2, 4, \ldots, 2n)$ are, respectively, the type-A and type-B Eulerian polynomials.

Visontai and I showed that *all* s-Eulerian polynomials, and therefore type-A and type-B, have only real roots. There were two key ideas involved in the proof. The first was to use a refinement of $E_n^{(s)}(x)$, natural for inversion sequences, that could be defined recursively as a sum of smaller such polynomials, some of which are multiplied by x. The second idea was to use the method of "compatible polynomials" of Maria Chudnovsky and Paul Seymour[18] to show that real rootedness of the sum followed from the real-rootedness of the smaller polynomials as long as the factor of x came in at just the right place (which, luckily, it does). In a section of his paper *Unimodality, log-concavity, real-rootedness and beyond*, Petter Brändén[19] has given a nice explanation of our method and generalized it to apply more broadly.

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

Savage: I think there is more to learn about lecture hall partitions. Here are a few examples below of directions since my 2016 survey paper.

Sylvie Corteel and Jang Soo Kim[20] recently discovered a two-dimensional version, *lecture hall tableaux*, with a surprisingly simple product form generating function. The generating function arises as the moment of certain orthogonal polynomials that Corteel and Kim were studying.

Petter Brändén and Madeleine Leander[21] have a recent paper on *lecture hall P-partitions*. I think Brändén and Leander's methods could be used to derive Corteel and Kim's formula for bounded lecture hall tableaux, but I haven't figured out how yet.

Brändén and Liam Solus[22] have a recent paper on the algebraic properties of lecture hall polytopes.

Also, the s-inversion sequences can be viewed as a combinatorial generalization of permutations. They can be used to interpret, refine, and generalize formulas in the same way that structures such as partitions and permutations might be used. For example, Mirko Visontai[17] and I used them to interpret the type-A and type-B Eulerian polynomials. More recently, they are being used to re-interpret and generalize results on pattern avoiding permutations.

Mansour: In a very recent pre-print, *Coefficients of the inflated Eulerian polynomials*[23], co-authored with Juan S. Auli and Ron Graham, you proved a conjecture of Pensyl and Savage by showing that the inflated s-Eulerian polynomials are unimodal for all choices of positive integer sequences s. Would you elaborate on this work and possible future research directions?

Savage: In the same way that s-Eulerian polynomials arise as h^*-polynomials of s-lecture hall polytopes $P_n^{(s)}$, the inflated s-Eulerian polynomials arise from the *rational* s-lecture hall polytopes, $Q_n^{(s)}$, consisting of all points $\lambda \in P_n^{(s)}$ with $\lambda_n \leq 1$. Thomas Pensyl and I showed that the inflated s-Eulerian polynomial[24] has the explicit expression

$$Q_n^{(s)}(x) = \sum_{e=(e_1,\ldots,e_n)\in I_n^{(s)}} x^{s_n asc(e)-e_n}.$$

[18]M. Chudnovsky and P. Seymour, *The roots of the independence polynomial of a clawfree graph*, J. Combin. Theory Ser. B 97:3 (2007), 350–357.

[19]P. Brändén, *Unimodality, log-concavity, real-rootedness and beyond*, Handbook of enumerative combinatorics, 437–483.

[20]S. Corteel and J. S. Kim, *Lecture hall tableaux*, Adv. Math. 371 (2020), 107266.

[21]P. Brändén and M. Leander, *Lecture hall P-partitions*, J. Comb. 11:2 (2020), 391–412.

[22]P. Brändén and L. Solus, *Some algebraic properties of lecture hall polytopes*, Sém. Lothar. Combin. 84B (2020), Article 25.

[23]J. S. Auli, R. Graham, and C. D. Savage, *Coefficients of the inflated Eulerian polynomial*, arXiv:1504.01089.

[24]T. W. Pensyl and C. D. Savage, *Rational lecture hall polytopes and inflated Eulerian polynomials*, Ramanujan J. 31 (2013), 97–114.

The coefficients of $Q_n^{(s)}(x)$ seemed to form a unimodal sequence for all s, but we could not prove it.

Later, I saw similar polynomials

$$T_n(x) = \sum_{\pi \in S_n} x^{n\ des(\pi)-(n-\pi_n)}$$

appearing in a paper by Fan Chung and Ron Graham[25]. One of the things Chung and Graham proved was that $T_n(x)/(1+x+\cdots+x^{n-1})$ and $T_{n-1}(x)$ have the same sequences of nonzero coefficients. Juan Auli and I found that this same property held for the inflated s-Eulerian polynomials when s is nondecreasing and we posted the result to the arXiv. Ron saw our preprint and said he thought he could prove the coefficients were unimodal, so we joined forces, and the paper you mentioned came to be.

Mansour: It is a fact that not many women follow a professional career in mathematics. It is a discussion around the globe on how to get more women into mathematics. What do you think about this issue? How do you compare the working conditions for women when you started your career and nowadays? What should be done in the next ten years to involve more women in mathematics?

Savage: As an undergraduate, I went to an engineering school where there were only a handful of women. All of the students, male and female, struggled through the same freshman courses. The feeling of a shared experience stayed with me through graduate school and as a faculty member in a relatively new department trying to build its research programs.

However, I do not think I would have had a career in mathematics research without the kindness and encouragement of other mathematicians. I am especially grateful to Herb Wilf, Frank Ruskey, George Andrews, Sylvie Corteel, Donald Knuth, Fan Chung, and Richard Stanley for support, advice, and sometimes just saying the right thing at the right time. And there are so many others, may I keep going? My point here is that we all have a chance to make a difference - to make a female (or any) mathematician feel welcome, appreciated, respected.

Mansour: You are a fellow of AMS and SIAM. Election as a fellow is an honor bestowed upon members by their peers as distinguished for their contributions to the discipline. What do you think about the importance of being recognized by your fellows?

Savage: It is such a great honor to me. The recognition carries a lot of weight within my college. At the same time, it is humbling since I am well aware of other mathematicians who are at least as deserving.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Savage: I almost always start out by writing a little program to generate or count something. If an enumerative result emerges, I rely on basic techniques like recursion, generating functions, q-series identities, bijections to prove it. If that is not good enough, I try to learn whatever is required.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Savage: I am more likely to have a eureka moment when I see a pretty answer emerge - it lets you know you are finally asking the right question. Once you "see" the answer, proving it can be a challenge and require you to learn new things.

For example, we wondered how many lecture hall partitions[15] had Ferrers diagrams that would fit in an $n \times nt$ box. The answer, $(t+1)^n$, was quite a surprise and led to the discovery of the connections with Ehrhart theory.

As another example, if, instead of lecture hall partitions, you enumerate anti-lecture hall compositions A_n, i.e., integer sequences λ satisfying

$$0 \le \lambda_1/n \le \lambda_2/(n-1) \le \ldots \le \lambda_n/1$$

you find that[26]

$$\sum_{\lambda \in A_n} q^{\sum_{i=1}^n \lambda_i} = \prod_{i=1}^{n} \frac{1+q^i}{1-q^{i+1}}.$$

[25] F. Chung and R. Graham, *Inversion-descent polynomials for restricted permutations*, J. Combin. Theory Ser. A 120:2 (2013), 366–378.

[26] S. Corteel and C. D. Savage, *Anti-lecture hall compositions*, Discrete Math. 263(1-3) (2003), 275–280.

That was another surprise that led us to discover connections with other q-series identities.

I have gotten "aha" moments during Ira Gessel's talks. He explains things so clearly that I realize I can use his methods for one of the problems I am working on.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Savage: I return to the question of whether or not the Durfee polynomials $D_n(x) = \sum_d p(n,d)x^d$ are real-rooted whenever I learn something that might apply. But maybe their most interesting property is already settled. In the 1998 paper with Canfield and Corteel[27] where we showed that the coefficients of $D_n(x)$ are asymptotically normal as $n \to \infty$, we also showed that the most likely size of the Durfee square for a partition of n is asymptotic to $\frac{\sqrt{6}\log 2}{\pi}\sqrt{n}$. This means (as Alexander Yong[28] pointed out in his 2014 *Notices* article) that your h-index is very likely to be about 0.54 times the square root of your total number of citations.

Mansour: Professor Carla Savage, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

Savage: Thank you, Professor Mansour, for the opportunity and for such interesting questions.

[27]E.R. Canfield, S. Corteel, and C.D. Savage, *Durfee polynomials*, Elec. J. Comb. 5 (1998), R32.

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