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Interview with Michael Albert

Toufik Mansour


Photo by Sharon Pine

Michael Albert completed his undergraduate studies at the University of Waterloo in 1981. He obtained a DPhil from Oxford University in 1984 under the supervision of Hilary Priestley. He was a postdoc and assistant professor at the University of Waterloo from 1984 to 1987, then an assistant and associate professor at Carnegie Mellon University from 1987 to 1996. In 1996 he moved to Dunedin, New Zealand, where he is now a professor in the Department of Computer Science at the University of Otago.

Mansour: Professor Albert, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Albert: To me, combinatorics is that part of mathematics that deals with problems that, without too much suspension of disbelief, could be expressed in terms of questions about the manipulation, arrangement, specification, and description of discrete collections of physical objects.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Albert: Combinatorics has sometimes been, or seen itself as, the neglected stepchild of the rest of mathematics. More recently this has perhaps become a little less true, partly because of the growing realization that there is great depth in some areas of combinatorics, and also because of its centrality in the relationship between mathematics and computer science.

Mansour: What have been some of the main goals of your research?

Albert: I'm a problem-solver at heart, so my research has not been driven by any big goals. I guess if I were to say something it would be "to solve interesting problems, explain the solutions well, and to understand how they fit together". I am particularly interested in methods and techniques that solve whole families of related problems, rather than single instances.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Albert: As a child, I was apparently quite excited about doing arithmetic. We lived in Germany when I was four and five years old and did a lot of traveling by car. Apparently, I could be almost endlessly amused by simply being asked to do sums or multiplication problems. I was in an ordinary fourth-grade class in a small town in Ontario that seems to have had a remarkable level of mathematical aptitude (I know that at least five of us eventually got PhDs in mathematics) and we had a teacher who fostered a spirit of competition and excellence among us. In retrospect, I'm not entirely sure I approve of that, but at the time it was exciting and spurred us on.

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At my secondary school, the teachers had a very strong program of training for the various mathematics competitions and I had a fair bit of success there - I think that pretty much locked me into my path.

Mansour: Were there specific problems that made you first interested in combinatorics?

Albert: Not so much - I had done work in a variety of areas, but began collaborating with Alan Frieze at Carnegie-Mellon. After I moved to New Zealand, I got to know Richard Nowakowski[1] through the Mathematical Olympiad, and got interested in combinatorial game theory. Finally, when I joined the Computer Science department at Otago, Mike Atkinson introduced me to permutation patterns, which have been the main focus of my research since then.

Mansour: What was the reason you chose the University of Oxford for your Ph.D. and your advisor Hilary Ann Priestley?

Albert: I was awarded a Rhodes Scholarship which explains the Oxford choice. I do not think they quite knew what to do with me (and, to be fair, I did not quite know what to do with myself) and Hilary was suggested as my advisor, as someone who had a broad range of interests as well.

Mansour: What was the problem you worked on in your thesis?

Albert: I do not like to remember it, but if I must . . . My thesis was in category theory and, roughly speaking, categorized the underlying general principle behind the theorem "a category with products and equalizers has all limits". That is, I determined how to tell if, knowing that in a category limits of various types existed, whether or not limits of other types necessarily exist.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Albert: Most commonly a specific problem. But, my main focus is not necessarily on solving the problem but figuring out how the problem feels – that is, what general characteristics it might have that can be interpreted or used elsewhere.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Albert: Oh absolutely. My problem-solving style is very much to do nine wrong things and learn from your mistakes before getting it right. You can not be doing wrong things and not getting too discouraged unless you are pretty sure the result is correct. Of course, I also use a lot of computer experimentation to look at examples (and look for counterexamples) with an emphasis on trying to do so at a scale that coincidences arising due to small numbers are not likely to overwhelm the data. So, I often have the feeling when I start, that I have pretty strong evidence that something is true.

Mansour: What are the top three open questions in your list?

Albert: Whichever three problems I happen to be working on. Which, in the past couple of years has been difficult.

Mansour: What kind of mathematics would you like to see in the next ten to twenty years as the continuation of your work?

Albert: I do not think of it as the continuation of my work, but I think I would most like to see the continued expansion of the use of computing as an experimental tool in mathematics. I definitely do not mean the project of computer-generated proofs – that is an interesting area, but one I'm a bit skeptical about, though there seems to have been some advance recently in generating interesting (to humans) results.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Albert: Of course, there are core areas - they are what the community has decided belong in undergraduate degrees. And of course, some topics are more important than others – in particular all the mathematics required to do engineering, physics, chemistry, and science more generally, as well as that required to model our world and the people and relationships within it.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Albert: I think there is a clear distinction

[1] M. Albert and R. J. Nowakowski, *The game of End-Nim*, Electron. J. Combin. 8:2 (2001), Article R1.

but it is not what most people mean by the categories. Pure mathematics is more like Hardy's view of number theory, problems addressed simply because the solver judges them to be interesting or beautiful. Applied mathematics is just that - mathematics being applied to areas outside of mathematics itself. So to me, the study of e.g., properties of solutions to families of differential equations is pure mathematics, while the use of differential equations to model fluid flow is applied mathematics.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Albert: Do it only if it really is a vocation - if you feel strongly that it is something you must do. There are many, many professions available to people with strong mathematical and analytical skills.

Mansour: Would you tell us about your interests besides mathematics?

Albert: My main outside interests would be games (playing, watching, and thinking about), cooking, and travel.

Mansour: Besides your research activity, you have also taken part in the training of high schools students for International Mathematical Olympiads. What do you think about the role of such competitions to discover young aspiring mathematicians?

Albert: Since it is the system that discovered me, I naturally have a tendency to think positively of it. However, I do think that those of us involved in such programs need to recognize that they are, by their nature, more exclusionary than one would like to be the case, and to take positive actions to develop a more inclusive approach.

Mansour: In one of your highly cited papers coauthored with Atkinson, *Simple permutations and pattern restricted permutations*[2], you developed a general strategy for the enumeration of pattern restricted permutations by using the concept of simple permutations. Would you tell us about the main ideas of this paper and some other important results led by this paper?

Albert: I think the main idea here arose from the observation that some permutation classes had relatively simple recursive descriptions - starting with the famous class of 132-avoiding permutations. Using the symbolic method, when such a description exists then generally enumerative problems become simpler. So, the idea was to try and describe a reasonably general setting in which one could do that – and the simple permutations are in some sense the base of those recursive cases. So our result basically was "if you understand the simple permutations in a class, and how they expand, then you understand the class as a whole". We illustrated this largely with enumerative examples, but subsequently, others have used it in a wider variety of contexts (recognition, random generation, etc.).

Mansour: Some of your recent works are related to *prolific permutations* and *compositions*. Would you tell us about the main motivations behind these works and some possible future directions?

Albert: This is an interesting little area. The main base question is "Given a 'universe' U, for what sorts of pairs of structures A and B in U is it true that every expansion of B in U contains more copies of A than B did". In large universes this turns out to be quite restrictive – for instance, if U is the class of all simple graphs then it is immediate that only the singleton graph can be used as A (and then any B will do). And of course, in very small universes, all questions are easy (since the notion of structure and expansion is so restricted). So the main problem is to find some intriguing middle ground where this concept is not trivial.

Mansour: One interesting recent notion in pattern-avoiding permutation classes is *Wilf-collapse*. Would you tell us about it and list some interesting open questions?

Albert: The observation is that in many permutation classes, the enumeration sequences of subclasses are significantly restricted. A priori, if a permutation class has $f(n)$ permutations of size n there is no reason to suppose that the classes obtained by adding any single additional restriction from among those $f(n)$ should have the same enumeration sequence. But, frequently much more is true – indeed there are often $o(f(n))$ such enumeration se-

[2] M. H. Albert and M. D. Atkinson, *Simple permutations and pattern restricted permutations*, Discrete Math. 300:1-3 (2005), 1–15.

quences. The interesting question is to what extent this is a universal phenomenon, and why it occurs. The most concrete open question is whether it occurs in the 321-avoiding permutations.

Mansour: Researchers interested in permutation patterns can still not enumerate the class of permutations avoiding the pattern 1324. Why is it so difficult? Do you think that one day we will learn the corresponding Stanley-Wilf limit?

Albert: There is no reason to believe that any particular permutation class "should" have a nice structure. I think 1324 just happens to be the first example where we see this – it is so difficult just because it is perhaps the first 'typical' example. I think that, to all intents and purposes, we have the ability to approximate the limit to any degree of precision – either experimentally or through schemes that should in principle approximate it arbitrarily closely.

Mansour: PermLab[3] is a software developed by you for permutation patterns. Would you tell us about how researchers in the field can use it for their research? Do you update it from time to time?

Albert: I think it has been a tool widely used for its visualization through a, now somewhat dated, user interface. I have used the backend quite a bit experimentally, and others familiar with Java could do so as well. However, it has not been actively updated for some time. For newcomers to the field, I had suggested using Python (but make sure you understand the memory model if you intend to do this seriously). There is a package called 'permpy'[4] that contains many useful functions and some more extensive libraries are part of the larger 'combinatorial exploration' project at Reykjavik University.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Albert: During a sabbatical at St Andrews University, in 2004 I had been working (with Nik Ruškuc and Steve Linton[5]) on extending results on encoding permutations so that we could bring techniques from automata theory, or languages more generally, to bear on their enumeration and classification. The problem was interesting because there were a few different schemes of this type around, and earlier work we had done with Mike Atkinson[6] had exposed that even some quite simple regular cases seemed to be interesting. Anyhow, we had been exploring this idea of having 'slots' in the representation of an evolving permutation as places where entries might be added. But, there always seemed to be difficulties with non-uniqueness. At the time, my children were taking fencing lessons one night a week in a local school - I was waiting for them in the car musing about the problem and suddenly realized that changing the perspective from thinking of slots as places that could be filled to thinking of them as places that must be filled solved all the problems we had. The insertion encoding per se has not been terribly widely used (though I believe it could be) but the underlying idea has popped up again a few times (e.g., my work with Vince Vatter[7] on generating simple permutations in various classes).

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Albert: Not really.

Mansour: Do you see any differences in the mathematical research tradition between New Zealand and Europe/America?

Albert: It is not something I really think about - I had preferred to believe that the community as a whole today is much more global. Obviously, local conditions (specifically with respect to funding and employment opportunities) affect what and where things are done, but that is not really relevant to the discipline as a whole.

[3]See `http://www.cs.otago.ac.nz/PermLab/`.

[4]See `https://github.com/cheyneh/permpy`.

[5]M. Albert, S. Linton, and N. Ruskuc, *On the permutational power of token-passing networks*, In Permutation Patterns, St Andrews 2007, S. Linton, N. Ruškuc, and V. Vatter, Eds., vol. 376 of London Mathematical Society Lecture Note Series, Cambridge University Press, (2010), 317–338.

[6]M. Albert, M. D. Atkinson, and N. Ruškuc, *Regular closed sets of permutations*, Theoret. Comput. Sci. 306 (2003), 85–100.

[7]M. Albert and V. Vatter, *Generating and enumerating 321-avoiding and skew-merged simple permutations*, Electron. J. Combin. 20:2 (2013), #P44.

Mansour: We know that you are an avid bridge player and former president of the Otago Bridge Club. How did you become interested in bridge? Is there a particular reason why you chose bridge rather than another game such as chess?

Albert: I have more or less given up playing bridge in the last five years, but certainly I was once an avid player. I became interested in bridge because I'm interested in games requiring some level of logic and analysis more generally, but more importantly because there were four of us who used to go out for a beer at the graduate student pub in Waterloo, and two of them played bridge (so soon enough, the other two of us did too). I think the main reason I chose bridge over other games (it replaced go as my principal 'mind sport') was that I liked the fact that you had seven minutes (one hand) to solve a problem (or series of problems) and then could start fresh. While one learns a lot from losing a two-hour game of chess or go, one has also spent two hours doing so!

Mansour: In a very recent short article, published in the Newsletter of the European Mathematical Society, professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question "Who Owns the Theorem?" concluded that "Mathematical truths exist, and mathematicians only discover them." On the other side, there are opinions around that "mathematical truths are invented." As a third way, some people claim that it is both invented and discovered? What do you think about this old discussion?

Albert: For me, it is clear that the feeling is one of discovery, but I think it is a psychological, not a logical distinction. I would not argue with someone else who felt they were inventing them. What that should imply about 'ownership' is another question entirely of course, and one of politics and law. I feel quite strongly that the body of mathematical knowledge should be free for use by all and have found myself quite uncomfortable at times with people who feel they need to hoard or protect 'their' results.

Mansour: Professor Michael Albert, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.