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Interview with Andrei Raigorodskii

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Andrei Raigorodskii graduated with distinction from the Department of Mechanics and Mathematics of Lomonosov Moscow State University (MSU) in 1998. He completed doctoral studies at the Number Theory Department of the Faculty of Mechanics and Mathematics, MSU in 2001. In the same year, he obtained a Ph.D. for research on combinatorial geometric properties of point sets. In 2004, he received a higher doctorate degree (Sc.D.) in discrete mathematics and mathematical cybernetics for research on problems of Borsuk and Nelson–Erdős–Hadwiger. Since 2016 he is the head of the Laboratory of Advanced Combinatorics and Network Applications at MIPT and a professor of mathematics. His awards include the Prize of the Presidium of the Russian Academy of Sciences (2005) and the Russian President's Prize in Science and Innovation for Young Scientists (2011). Professor Raigorodskii gives lectures at the Moscow Institute of Physics and Technology (MIPT), Lomonosov Moscow State University, and the New Economics School (NES).

Mansour: Professor Raigorodskii, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Raigorodskii: Formally, combinatorics is a branch of mathematics, which is devoted to the study of different collections of objects and their interrelations. However, I understand quite well that the question is not about formal definitions. So I will proceed in another way. Being the director of Phystech-School of Applied Mathematics and Computer Science (it is a faculty at the Moscow Institute of Physics and Technology) and spending also a lot of time popularizing mathematics, I used to tell in my lectures that mathematics is beautiful not because it has applications, but it has applications because it is beautiful! From this point of view, combinatorics is the best part of mathematics to prove this assertion. A threshold, which has to be surpassed in order one understands combinatorial questions, is low enough, so that a high-school student is ready to follow combinatorial arguments. I also use the word "catharsis" to depict what sometimes happens after a combinatorial lecture.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Raigorodskii: Some people believe that combinatorics is just the "handmaid" of really "central" parts of mathematics such as algebraic geometry or topology. Some people think that combinatorics is just a good basis for computer science. However, during the last 100 years, combinatorics has become an absolutely independent field of mathematics, which is not only full of great and profound questions but also penetrated by very non-trivial meth-

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ods coming from algebraic geometry, topology, probability theory, dynamical systems, etc. For example, when László Lovász[1] solved Kneser's conjecture on the chromatic number of a graph, which is also named after Kneser, no one could assume that topology would have helped him!

Mansour: What have been some of the main goals of your research?

Raigorodskii: I'm not only a researcher. I also spend a lot of time popularizing mathematics, attracting people to the research, to organize a faculty, where the best students get involved in the world of deepest math. I'm 46 years old, and I already have 32 young people who got their Ph.D. under my supervision. Four of them got the highest Russian degree of doctor of sciences, which is an analog of habilitation in some countries. So my goal is every time not only in attacking hard problems but also in developing talents for making even more attacks on such problems from different sides.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Raigorodskii: My grandfather was a mathematician. He graduated from MSU, Mechanics and Mathematics Faculty, just before the World War II. After the war, he worked in the space industry. I loved him very much, and his influence was certainly very important. My parents both graduated from the Moscow Institute of Transport Engineers. My father is a programmer, and he is still working (he is 69 years old). My mother stopped working at the beginning of the 90s, but she spent a lot of time, when I was a child, demonstrating to me the beauty of mathematics. Anyway, I was capable enough.

Mansour: Were there specific problems that made you first interested in combinatorics?

Raigorodskii: A somewhat stupid, but the important problem was as follows. In a mathematics classroom of my primary school, there was (of course) a blackboard. On a part of this blackboard, a grid was drawn consisting of equal squares. I was wondering whether it was possible to calculate quickly the number of different squares in this grid (not only the smallest ones but also those with 2 smallest squares on every side, etc). My grandfather explained to me a general formula ($1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$), and I spent a lot of time in producing analogous general formulas for other figures that could be calculated inside a grid $n \times n$.

Mansour: What was the reason you chose the Faculty of Mechanics and Mathematics (MSU) for your Ph.D. and your advisor?

Raigorodskii: In the 90s, there were only two faculties, where one could study pure mathematics in Moscow: Mechanics and Mathematics (Mechmath) Faculty and Faculty of Computational Mathematics and Cybernetics. Both faculties were in MSU. They are still there. However, now there are many other opportunities in Russia to study pure mathematics and applications including our Physech School of Applied Mathematics and Informatics. Unfortunately, for MSU, new faculties are much more attractive. But 30 years ago, I was choosing among two variants. I finally chose Mechmath Faculty, since it was more oriented toward pure mathematics.

As for my advisor, it was a really great choice. I believe that if I did not do it, I would probably not have become the mathematician who gives you this interview. My advisor was Nikolay Germanovich Moshchevitin. Now he is a well-known specialist in number theory. But in 1995, when I chose to go to the Department of Number Theory of Mechmath Faculty, Nikolay Germanovich was just a 28 year old assistant professor. A strange choice? No! Nikolay Germanovich was very enthusiastic, and he was the main motivator of my real scientific work. Due to him, I wrote my first paper[2] between my second and third years of studies at Mechmath Faculty (1995), and published a counterexample[3] to Borsuk's conjecture in 1997, when I was in the 4th year of my studies. Nikolay Germanovich was also the scientific advisor of Ilya Shkredov, who is a renowned specialist in additive combinatorics.

[1] L. Lovász, *Kneser's conjecture, chromatic number and homotopy*, J. Combinatorial Theory Ser. A 25 (1978), 319–324.

[2] A. M. Raigorodskii, *Systems of common representatives*, Fundamentalnaya i Prikladnaya Matematika 5 (1999), N3, 851–860 (in Russian).

[3] A. M. Raigorodskii, *On dimensionality in the Borsuk problem* (Russian) Uspekhi Mat. Nauk 52 (1997), no. 6(318), 181–182; translation in Russian Math. Surveys 52:6 (1997), 1324–1325.

Mansour: What was the problem you worked on in your thesis?

Raigorodskii: First, it was a problem on the edge of the geometry of numbers and combinatorics. By a lattice in $\mathbb{R}^n$, we mean a point set Λ, which is an integer span of a set of linearly independent vectors. Consider a lattice Λ containing $\mathbb{Z}^n$. What is the difference between the standard orthonormal basis of $\mathbb{Z}^n$ and a basis in Λ? A way of calculating this difference was proposed by Moshchevitin. I do not want to go into the details here, but I completely solved the problem[4] in some important cases: for example, in the case when the factor $\Lambda/\mathbb{Z}^n$ is a cyclic group. To this end, I found new lower bounds for piercing numbers of some families of sets.

Second, it was a famous problem in combinatorial geometry — Borsuk's problem concerning the minimum number of parts of smaller diameter, into which an arbitrary bounded set in $\mathbb{R}^n$ can be partitioned. I found a new counterexample[3] to Borsuk's conjecture stating that the number should be $n + 1$, and I also obtained a new general subexponential lower bound[5] for Borsuk's number.

Surprisingly, in Borsuk's problem, the techniques of piercing numbers for families of sets could also be applied[6].

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Raigorodskii: I definitely prefer concrete statements.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Raigorodskii: Yes, I do. Intuition is very important!

Mansour: What are the top three open questions on your list?

Raigorodskii: First, Borsuk's conjecture and the chromatic numbers of metric spaces[7]. Second, conditions on "stability" of the independence numbers and the chromatic numbers of random subgraphs in sequences of graphs[8]. Third, general results on sparse pseudorandom graphs[9].

Mansour: What kind of mathematics would you like to see in the next ten to twenty years as the continuation of your work?

Raigorodskii: Probably I do not understand quite well the question. Since I have a lot of very successful former students, I understand that they will further develop our joint directions of work — in combinatorial geometry, extremal and probabilistic combinatorics, graph and hypergraph theories, random structures and algorithms, web graph modeling, etc. In general, I hope my students will surpass me and will obtain great results in really good mathematics!

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Raigorodskii: I do not think so. I believe that in every area there are essential problems, which are very important to understand "how the world is created", and of course, in every area, one can be engaged in nonsense.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Raigorodskii: In some sense, I have already answered this question: "Mathematics is not beautiful because it has applications, but it has applications because it is beautiful." "Applied" mathematics does not exist without "catharsis", which appears as a result of real discovery. I myself worked a lot in both "parts", and I see that actually, the difference is not so great. When I was creating in 2008 the first research division at Yandex (Google is an American

[4] A.M. Raigorodskii, *The defects of admissible balls and octahedra in a lattice, and systems of generic representatives*, Mat. Sbornik 189 (1998), N6, 117–141; English transl. in Sbornik Math. 189 (1998), N6, 931–954.

[5] A.M. Raigorodskii, *On a bound in Borsuk's problem*, Uspekhi Mat. Nauk 54 (1999), N2, 185–186; English transl. in Russian Math. Surveys 54 (1999), N2, 453–454.

[6] A. M. Raigorodskii, *The Borsuk and Grünbaum problems for lattice polytopes*, Izvestiya of the Russian Acad. Sci. 69 (2005), N3, 81–108; English transl. in Izvestiya Math. 69 (2005), N3, 513–537.

[7] A. M. Raigorodskii, *The Borsuk problem and the chromatic numbers of some metric spaces*, (Russian) Uspekhi Mat. Nauk 56 (2001), no. 1(337), 107–146; translation in Russian Math. Surveys 56:1 (2001), 103–139.

[8] L. I. Bogolyubskii, A. S. Gusev, M. M. Pyaderkin, and A. M. Raĭgorodskiĭ, *Independence numbers and chromatic numbers of random subgraphs in some sequences of graphs*, Dokl. Math. 90 (2014), 462–465.

[9] A theory of pseudorandom graphs is well developed for dense graphs. See, for example, `https://arxiv.org/pdf/math/0503745.pdf`. In the case of sequences of sparse graphs (when the number of edges is much smaller than the square of the number of vertices) - for example, in the case of growing web graphs - an analogous theory is not yet developed

Yandex), our first result was rough as follows. We proposed a probabilistic model[10,11] of the Web-graph growth. Then we proved several quite non-trivial "purely mathematical" theorems concerning the model. We also made a careful statistical analysis "tuning" some parameters of the model so that it became as "realistic" as possible. Finally, we created a feature for learning to rank improving significantly the quality of the Yandex search engine. Was it pure or applied math?

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Raigorodskii: As long as you can do so, please motivate yourselves with "catharsis", with "I love doing that" and not with "it is important for real-life" or "it will give me more money"! It is really important to make such "investments". In other cases, you will have a ceiling above which you will not be able to jump.

Mansour: Would you tell us about your interests besides mathematics?

Raigorodskii: I love traveling. In 1999, I spent 37 days traveling around Europe with the help of Interrail ticket, which allowed me to enter any train (second class coach). In 2000, I did the same in the US with the help of Ameripass (traveling for 30 days in Greyhound buses). In 2004, I spent 42 days driving with my friends in a Russian Volga car from Moscow to Vladivostok and back (22500km in total), and so on.

Mansour: Besides your research activity, you use your time to reach out to school children through lectures. What is your primary motivation for these activities? Working on new ideas and explaining them to non-professionals requires very different efforts. How do you compare these two activities?

Raigorodskii: I believe that teaching is at least as much important as doing research is. Of course, not everyone likes teaching. Moreover, the majority of researchers are unable to transfer their knowledge to a broad collection of people. Yet being a student, I realized that my lectures given at various seminars seem to be very transparent as well as motivating for those who listened to them. And I had a dream to ignite as many people as possible with mathematics.

Explaining ideas to non-professionals is really hard, but very delightful! When you see how the listeners' eyes light up, it's already you who feels catharsis.

I give around 8 lectures every week on average, 4-5 of them are for students of MIPT and some other universities, and 3-4 of them are for high-school students or just for a broad audience. As I love traveling, I visit about 20 different cities in Russia per year with such lectures. Since I do research at the same time as I attend to my organizational duties for the Phystech-School of Applied Mathematics and Informatics, I typically fly out of Moscow on a Thursday evening and return Monday evening. During these 4 days, I visit four cities (e.g. X,Y,Z, and W) and deliver 2-3 lectures in each.

Mansour: During 62nd International Mathematical Olympiad (IMO 2021), participants had the opportunity to attend *Exclusive Interview of the Day*, in which you had a lecture on *random graphs*. How different were these lectures from regular olympiad training lectures? Did you receive any intelligent questions from participants? Some math olympiad problems lead to research problems and vice versa. Would you share your experience with us, if any?

Raigorodskii: Usually, I do not popularize standard mathematical facts, which is done by many other people who also popularize mathematical knowledge. Almost all my lectures of this kind are devoted to subjects, on which I myself did research. For example, one of the most popular lectures that I love giving is about Borsuk's problem. Young people are fond of it! And not only young.

Of course, my lecture on random graphs[12] was also far enough from the format of training lectures. I spoke about the history and modern trends in the theory of random structures. But the listeners responded well to this level

[10] E. A. Grechnikov, G. G. Gusev, L. A. Ostroumova, Yu. L. Pritykin, A. M. Raigorodskii, P. Serdyukov, D. V. Vinogradov, and M. E. Zhukovskiy, *Empirical validation of the Buckley–Osthus model for the web host graph*, The 21st ACM Conference on Information and Knowledge Management, 2012, 1577–1581.

[11] E. A. Grechnikov, *The degree distribution and the number of edges between nodes of given degrees in directed scale-free graphs*, Internet Math. 11 (1015), No. 6, 487–527.

[12] See `http://ecajournal.haifa.ac.il/Volume2023/ECA2023_S3I4_slides1.pdf`.

and did really ask good questions.

As for the confluence between problems in real research and in math olympiads, I myself composed several problems for Moscow Math Olympiad coming from my personal research experience. One of them is as follows. Consider $4n$ points on the plane. Join two points by a segment, if the distance between them equals 1. Assume that among any $n+1$ points, one has at least one segment. Prove that the total number of segments is at least $7n$. It was a hard problem, but one of the high-school students succeeded in solving it. Is it all that happened? No! From this problem, I made a very popular lecture trying to attract people to this research area. Indeed, $7n$ was obviously not the best possible bound. Eventually, I found a student, who first improved this bound to $8n$ and then to $26n/3$. The last result is quite non-trivial. It was published in 2016 in Discrete and Computational Geometry[13], which is one of the best journals in its field.

Mansour: You are the author of 20 books and brochures. One of them is Combinatorics and Probability Theory (a study guide for the School of Data Analysis (DA)). How does combinatorics enter into the picture of data science? Some researchers consider data science as a new revolution in science. What do you think about this new field and its interaction with other branches of mathematics?

Raigorodskii: DA cannot live without combinatorics and probability. Of course probability itself has a combinatorial background. But combinatorics is the real base of DA. First of all, there are graphs, which are obviously used everywhere. Then, there are hypergraphs that generalize graphs: here every edge may consist of several vertices, not of just two. For example, if you study collaboration networks, you get much more information about them when you do not just make an edge with two vertices, if the corresponding authors share a paper, but produce a hyperedge consisting of all the co-authors of a given paper. On the other hand, one can recall the notion of the so-called VC-dimension, which is intensively studied by both combinatorialists and data scientists. This notion goes back to Russian mathematicians Vapnik and Červonenkis[14], who invented it at the beginning of the 70s as an instrument to prove uniform convergence in laws of large numbers. Generally speaking, VC-dimension is attributed exactly to hypergraphs and their infinite analogs.

DA uses not only combinatorics and probability. To understand modern tools of DA, you should deeply know topology and higher algebra, random structures, and algorithms.

Mansour: In one of your highly cited papers *Borsuk's problem and the chromatic numbers of some metric spaces*[7], among others, you considered the problem of finding chromatic numbers of some metric spaces. Would you explain how *chromatic numbers* are connected to *metric spaces*?

Raigorodskii: Yes! It is related to the math olympiad problem I described above. The main question is in finding the minimum number of colors needed to color all the points in a metric space so that any two points at a distance from a given set of positive reals get different colors. This number is called the chromatic number of the metric space. If your metric space is the Euclidean plane and a set of reals consists of just one number 1, then the coloring problem becomes the problem of finding the chromatic number of a graph, whose vertex set coincides with $\mathbb{R}^2$ and whose edges are all possible pairs of points lying at distance 1. De Bruijn and Erdős[15] proved in 1951 that the chromatic number of such an infinite graph, being finite, is attained on a finite subgraph. So the math olympiad problem is about possible bounds for the number of edges in graphs, whose chromatic numbers could be equal to the chromatic number of the Euclidean plane. The question[16] of determining this chromatic number is still open! We only know that the chromatic number[17] lies between 5 and 7.

[13] L. E. Shabanov and A. M. Raigorodskii, *Turán type results for distance graphs*, Discrete Comput. Geom. 56:3 (2016), 814–832.

[14] V. N. Vapnik and A. Ya. Červonenkis, *On uniform convergence of the frequencies of events to their probabilities*, Theory Probab. Appl. 16:2 (1971), 264–280.

[15] N. G. De Bruijn and P. Erdős, *A colour problem for infinite graphs and a problem in the theory of relations*, Nederl. Akad. Wetensch. Proc. Ser. A 54 (1951), 371–373.

[16] A. M. Raigorodskii, *Coloring distance graphs and graphs of diameters*, Thirty Essays on Geometric Graph Theory, J. Pach ed., Springer, 2013, 429–460.

[17] A. D. N. J. De Grey, *The Chromatic number of the plane is at least* 5, arXiv:1804.02385v3.

[18] A. M. Raigorodskii, *On the chromatic numbers of spheres in* $\mathbb{R}^n$, Combinatorica 32:1 (2012), 111–123.

Mansour: In your paper[18] *On the chromatic numbers of spheres in* $\mathbb{R}^n$, you studied the quantity $\chi(S_r^{n-1})$, the minimum number of colors needed to color the points of a sphere S_r^{n-1} of radius $r \geq \frac{1}{2}$ in $\mathbb{R}^n$ so that any two points at the distance 1 apart receive different colors. You showed that for every $r > \frac{1}{2}$ the quantity $\chi(S_r^{n-1})$ grows exponentially, not linearly, contrary to what was considered before. Would you tell us about the main ideas behind this result?

Raigorodskii: To prove this result, we have to find a distance graph, whose vertices lie on the sphere S_r^{n-1} and whose edges are segments of length 1. We consider graphs with vertex sets consisting of (0,1)-vectors or of (-1,0,1)-vectors. Of course, we normalize these vectors in the order they fall onto the sphere. Then, for such vectors, to be at distance 1 is equivalent to have some concrete scalar product. We use the bound $\chi(G) \geq |V|/\alpha(G)$, where $\alpha(G)$ is the independence number of G, i.e., the maximum number of vertices (vectors), which are pairwise disjoint. In turn, to estimate α we use a linear algebra method. Namely, to each vector, we assign a polynomial with coefficients in a finite field, and then we prove that the polynomials assigned to an independent set of vectors are linearly independent over their field, which means that their number is at most the dimension d of a space, in which the polynomials lie. Finally, we calculate the asymtotics of the ratio $|V|/d$ and see that it is exponential in n.

Mansour: In one of your very recent papers[19], coauthored with M. M. Koshelev, *New bounds on clique-chromatic numbers of Johnson graphs*, you significantly improved lower bounds of the clique-chromatic number $\chi_c(G)$ of a graph G for some families of Johnson graphs. Would you tell us more about this work?

Raigorodskii: By Johnson's graphs we mean the graphs $G(n, r, s)$. Here the set of vertices consists of all possible r-subsets of an n-set, and the set of edges is formed by all possible pairs of vertices, which correspond to sets having exactly s elements in common. Equivalently, vertices are n-dimensional (0,1)-vectors, and their scalar product s is responsible for drawing an edge. Such graphs are very important. They appear in coding theory[20], since their independent sets are codes with one forbidden distance. Moreover, the maximum cliques in graphs $G(4k, 2k, k)$ are easily translated into Hadamard matrices. Johnson's graphs are also used to make lower bounds for the chromatic numbers of metric spaces that we have already discussed. They give counterexamples to Borsuk's conjecture that we have discussed and will discuss again below. They help producing explicit lower bounds for Ramsey numbers[21]. The graphs $G(n, r, 0)$ are called Kneser's graphs, and we have mentioned them when speaking about Lovász's unexpected break-through (he succeeded in proving[22], by using algebraic topology, that the chromatic number of $G(n, r, 0)$ equals $n-2r+2$, provided $2r \leq n$).

In our works with Koshelev (and, in fact, with D. A. Zakharov earlier), we studied another coloring problem for $G(n, r, s)$. It is concerned with the so-called clique chromatic numbers. In this case, we are seeking the minimum number of colors needed to color all vertices so that every maximal (by inclusion) clique with at least 2 vertices gets at least 2 different colors. The notion of clique-chromatic number is very different from that of classical chromatic number, since a subgraph of a graph may have a much bigger clique-chromatic number than the graph itself. Indeed, a complete graph has clique-chromatic number 2. However, all of us know a classical Erdős result stating that there exists a graph with high girth and high chromatic number. But if a graph has girth greater than 3 (does not contain triangles), then its clique-chromatic number coincides with its usual chromatic number.

In our works, we found a relation between clique chromatic numbers of Johnson's graphs with hypergraph multicolor Ramsey numbers, and so we got new bounds substantially improving the previously known ones.

[19] A. M. Raigorodskii and M. M. Koshelev, *New bounds on clique-chromatic numbers of Johnson graphs*, Discrete Appl. Math. 283 (2020), 724–729.

[20] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*, North-Holland, Amsterdam, 1977.

[21] P. Frankl and R. Wilson, *Intersection theorems with geometric consequences*, Combinatorica 1 (1981), 357–368.

[22] J. Matoušek, *Using the Borsuk-Ulam theorem. Lectures on topological methods in combinatorics and geometry*, Written in cooperation with Anders Björner and Günter M. Ziegler. Universitext. Springer-Verlag, Berlin, 2003.

Mansour: You have a series of papers on *Borsuk's problem*. Would you elaborate on the problem and your related works?

Raigorodskii: Oh... I can speak about Borsuk's problem for an arbitrarily long period of time. I have several surveys[16] published between 2001 and 2016. I have several books popularizing this fantastic story. OK, let me just briefly explain what happened.

In 1933, Borsuk[23] proved that any set of diameter 1 in the plane can be partitioned into 3 parts of smaller diameter. The proof uses a notion of universal covers. Namely, one can show that a regular hexagon with a distance of 1 between parallel sides covers any set of diameter 1 (this is called the "lemma of Pál"[24]). Although the hexagon itself has diameter greater than 1, it can readily be divided into three equal parts of diameter $\frac{\sqrt{3}}{2} < 1$. On the other hand, the 3 vertices of a regular triangle show that there are sets on the plane that cannot be divided into two parts of smaller diameters.

Borsuk conjectured that in any dimension, the minimum number $f(n)$ of parts of smaller diameter needed to partition an arbitrary set of diameter 1 equals $n+1$. It was a very important topological conjecture which could relate dimension with discrete geometry.

The conjecture was proved[25,26] for $n \leq 3$. A lot of people made efforts to prove it in higher dimensions. The conjecture was proved for spheres (Borsuk), for smooth convex bodies (Hadwiger[27,28]), for somehow symmetric sets (Rogers et al.[29]), but not even for finite sets or, equivalently, for polytopes. Some upper bounds on $f(n)$ appeared. Among them, $f(n) \leq 2^{n-1} + 1$ (Lassak[30]) and $f(n) \leq \left(\sqrt{\frac{3}{2}} + o(1)\right)^n$ (Schramm; Bourgain and Lindenstrauss[31,32]). Terribly far from the expected linear growth!

In 1993, Kahn and Kalai[33] suddenly disproved Borsuk's conjecture! They used a purely combinatorial result of Frankl and Wilson, which can be formulated, roughly, as a bound on the independence number of Johnson's graph. So the counterexample proposed by Kahn and Kalai was a finite set of (0,1)-vectors! The proof consisted of just 10 lines or something like that. Only one "little" problem: counterexamples worked for $n \geq 2015$, but not for smaller dimensions.

Real competition has opened to dominate the dimension of a counterexample. I succeeded in proving in 1997 that $f(n) > n+1$ for $n \geq 561$. Now a record is due to Bondarenko: Borsuk's conjecture is wrong[34,35] for $n \geq 64$.

Nobody knows what happens for $n \in [4, 63]$. I would guess that for $n = 4$ the conjecture is true, and it fails already for $n \geq 5$. Maybe it fails even for $n = 4$.

Another direction is in bounding $f(n)$ from below. Now the record is my bound[36]

$$f(n) \geq \left(\left(\frac{2}{\sqrt{3}}\right)^{\sqrt{2}} + o(1)\right)^{\sqrt{n}} = (1.2255 \cdots + o(1))^{\sqrt{n}}$$

made in 1999.

You can see that the difference between upper and lower bounds is still huge, but not so terrific as it was in times when almost every specialist believed that the conjecture must be true.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

[23] `http://www.atractor.pt/mat/GeomConv/borsuk-_en.html`.

[24] J. Pál, *Über ein elementares Variationsproblem*, Danske Videnskab. Selskab., Math. Fys. Meddel. 3:2 (1920), 35.

[25] K. Borsuk, *Drei Sätze über die n-dimensionale euklidische Sphäre*, Fundamenta Mathematicae , 20 (1933), 177–190.

[26] J. Perkal, *Sur la subdivision des ensembles en parties de diamètre inférieur*, Colloquium Mathematicum 2 (1947), 45.

[27] H. Hadwiger, *Überdeckung einer Menge durch Mengen kleineren Durchmessers*, Commentarii Mathematici Helvetici 18(1) (1945), 73–75.

[28] H. Hadwiger, *Mitteilung betreffend meine Note: Überdeckung einer Menge durch Mengen kleineren Durchmessers*, Commentarii Mathematici Helvetici 19(1) (1946), 72–73.

[29] C. A. Rogers, *Symmetrical sets of constant width and their partitions*, Mathematika 18 (1971), 105–111.

[30] M. Lassak, *An estimate concerning Borsuk partition problem*, Bull. Acad. Polon. Sci. Sér. Sci. Math. 30 (1982), 449–451.

[31] J. Bourgain and J. Lindenstrauss, *On covering a set in RN by balls of the same diameter, In: Geometric Aspects of Functional Analysis*, Lecture Notes in Math., 1469, Springer-Verlag, 1991, 138–144.

[32] O. Schramm, *Illuminating sets of constant width*, Mathematika 35(2) (1988), 180–189.

[33] J. Kahn and G. Kalai, *A counterexample to Borsuk's conjecture*, Bull. Amer. Math. Soc. 29(1) (1993), 60–62.

[34] See `https://www.researchgate.net/publication/236687922_On_Borsuk's_Conjecture_for_Two-Distance_Sets`.

[35] T. Jenrich and A. E. Brouwer, *A 64-Dimensional Counterexample to Borsuk's Conjecture*, Electron. J. Combin. 21(4) (2014), #P4.29.

[36] A. M. Raigorodskii, *On a bound in the Borsuk problem* (Russian) Uspekhi Mat. Nauk 54 (1999), no. 2(326), 185–186; translation in Russian Math., Surveys 54:2 (1999), 453–454.

Raigorodskii: The majority of my works are devoted to extremal combinatorics, and so I mostly use algebraic, probabilistic, or topological methods. Sometimes enumeration techniques also help me. For example, they appear in random graph subjects when it is important to calculate or bound somehow the numbers of concrete subgraphs in a graph.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Raigorodskii: I am permanently thinking about a question. Even when I sleep. Of course, the impressions of a young researcher are novel and more powerful. The story of my counterexample to Borsuk's conjecture is actually very interesting and somehow dramatic. In 1997, it was already known that the conjecture was false for $n \geq 903$. After a month of constant meditations, I found a counterexample for $n = 630$. This result was considered by many specialists including my supervisor Moshchevitin as a great achievement and we started to prepare a paper. Simultaneously, there was a "referee" who tried to find a mistake in my proof (even before a paper was written and submitted). I gave a lot of lectures at different seminars. Finally, all of us decided that there were no mistakes and that the paper should be submitted as soon as possible. Just one more seminar, and go! At this last seminar, there was a student (I knew him). At some point, he asked me to explain in detail a part of an argument. We started to discuss it and suddenly found a very subtle bug. The proof was destroyed. I think you can imagine what I felt. After that, I could do nothing! I could only think about Borsuk. It took one more week. I had to find several new ideas, and finally, I got a counterexample for $n = 561$. This result was true. It was not just a "eureka". It was a real "catharsis"! Moshchevitin told me ironically: "Probably we have to find one more mistake in the order you further decrease dimension?" But there were no mistakes.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Raigorodskii: I think my favorite problems are that of Borsuk and of the chromatic numbers. I consistently come back to them. In total, my students and I have proved many different results concerning them.

Mansour: Do you see any differences in the mathematical research tradition between Russia and Europe/America?

Raigorodskii: Right now, it is not so clear. However, I would say that in general, a Russian is more inclined to dig into one subject for a long time. Sometimes it can be even considered as "boredness". In Europe — also "on average" — people are more inclined to often change subjects, to "take it easier". But this is just my "flair". Many counterexamples.

Mansour: Russians have a strong tradition in science, mathematics, chess, and other creative areas but not in philosophy (if I am correct)? You have many great novelists, mathematicians, physicists, and chess players but only a few very famous philosophers. How do you explain this?

Raigorodskii: In Russia, we had interesting philosophers before the revolution of 1917: Solovyev, Rozanov, Berdyaev, Bulgakov, and many others. Some of them emigrated and continued their work. Some of them are really great. Do not forget Dostoevskiy who can also be considered a philosopher! Some are naive and did not influence anything outside Russia. I think Russia just did not have enough time to develop philosophy. It is our tragedy, but every tragedy must be followed by a catharsis!

Mansour: In a very recent short article, published in the newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question "Who Owns the Theorem?" concluded that "Mathematical truths exist, and mathematicians only discover them." On the other side, there are opinions that "mathematical truths are invented." As a third way, some people claim that it is both invented and discovered? What do you think about this old discussion?

Raigorodskii: I follow here the same philosophy as Melvyn B. Nathanson. There is an ideal world, in which everything is done. A mathematician succeeds in taking ideas (results) from this world.

Mansour: My last question is philosophical: have you figured out why we are here?

Raigorodskii: I think today I preferred to stay philosophically silent.
Mansour: Professor Andrei Raigorodskii, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.