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Interview with Anne Schilling

Toufik Mansour

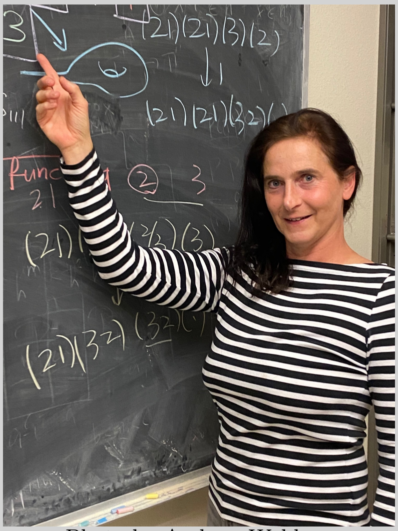

Photo by Andrew Waldron

Anne Schilling studied mathematics and physics at the University of Bonn. She went to the State University of New York at Stony Brook as a Fulbright scholar and completed her Ph.D. in 1997 under the supervision of Barry M. McCoy. From 1997 until 1999 she was a postdoctoral fellow at Institute for Theoretical Physics at Amsterdam University. From 1999 until 2001 she was a C.L.E. Moore Instructor in the Mathematics Department at the Massachusetts Institute of Technology. After that she joined the faculty of the Department of Mathematics at the University of California at Davis, where she is a Full Professor. Anne Schilling was a Humboldt Fellow in 2002, a Simons Research Fellow during 2012/13, and was elected to the 2019 class of fellows of the American Mathematical Society. Her research interests include Algebraic Combinatorics, Representation Theory, and Mathematical Physics. She is a member of the editorial board for various journals, notably the open-access journals Combinatorial Theory and Algebraic Combinatorics.

Mansour: Professor Schilling, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Schilling: Thank you for the invitation to this interview. It is an honor.

Combinatorics is the art of counting. It pops up in many different settings such as mathematics, computer science, engineering, and physics. I first encountered combinatorics when I studied physics, in particular, counting how many particles are in a given energy state. Viewing the particles as bosons versus fermions gives rise to different ways of counting them. This is one way of viewing the famous the Rogers–Ramanujan identities[1].

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics and physics?

Schilling: Combinatorics is often a common language that unites different areas in mathematics and physics. When a problem is translated into a combinatorial statement, it is relatively easy to phrase (which does not mean that it is easy to solve).

Mansour: What have been some of the main goals of your research?

Schilling: The main goal of my research is to understand mathematical structures better. Even if certain problems have already been "solved" it does not mean that I necessarily understand them. If a particular problem presents itself, I am really trying to get to the bottom of it. I sometimes describe the process as a random walk, it is not always a straight line to a solution, but a path that is not always predictable. On the way, I discover or understand many new things I did not even know existed. In this sense, you might say that the main goal of my research is to find

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[1] G. H. Hardy, *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work*, 3rd ed. New York: Chelsea, pp. 13 and 90-99, 1999.

as many connections between different areas of science as possible and unravel them. So far, I have started in mathematical physics and drifted to q-series and special functions, combinatorics and representation theory, semigroups, and Markov chains. I am curious to find out where else this path will take me!

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Schilling: Let me describe three scenes that are still in my memory as defining moments in my formative years:

Scene 1: I was done with my mathematics exam at school and handed in the paper early. My teacher Herr Schnur made me sit down again and asked me to relate the law of cosines $c^2 = a^2+b^2-2ab\cos\alpha$ that was on the exam to the Pythagorean formula. This took me a bit by surprise since I had not thought of these two formulas as being related. In hindsight, this is of course trivial since $\cos\alpha$ is zero for a right angle, but discovering this on my own as a child during the exam and seeing how topics in school are related to each other opened my eyes.

Scene 2: The scene is the beaches on the Atlantic coast of France. We are on summer holidays and I am 15 years old. My father writes the fundamentals of calculus in the sand. This is not like anything I had seen at school before. I soak up his scribbles before the waves wash the information away again. At night, I write up what I have understood and get hooked by the beauty of how mathematics is able to capture and describe nature.

Scene 3: It is my first semester as a student in physics and mathematics at the University of Bonn in Germany. Professor Hirzebruch has given us eight challenge problems over the Christmas break in our analysis lecture. I pull my hair out over the break to try to solve them and get pretty stuck on most of them. I manage to find some neat solutions to three of the problems and make partial progress on a fourth problem. It is the first day back after the break and everyone is talking about how they solved all or most of the problems. I do not understand most of their approaches, but they all seem to have nailed them. Reluctantly I hand in my 3 1/2 solutions. A week later Professor Hirzebruch comes back to class with two champagne bottles. One of them is for me! 'Your 3 1/2 solutions are quite original', he said. It turns out that most of the other students' solutions were wrong. I am glad that I did not get intimidated earlier to hand in my solutions despite all my doubts.

This last story made it into the Story Collider, see minute 18.20[2]

Mansour: Were there specific problems that made you first interested in combinatorics?

Schilling: When I was a graduate student in Stony Brook, generalizations of the Rogers–Ramanujan identities came out of the analysis of statistical mechanical models by studying so-called fermionic formulas[3]. These formulas can be interpreted in terms of crystal bases on the one hand and rigged configurations on the other hand. This begs for a bijection between these two combinatorial sets.

Mansour: What were the main reasons you chose Stony Brook University for your Ph.D. and your advisor Barry M. McCoy?

Schilling: When I was in Bonn as a student, I wanted to get some overseas experience and applied for a Fulbright scholarship. Fulbright sent me to SUNY Stony Brook for a year. I met my husband there and as a result, stayed longer than my exchange year. So I guess Stony Brook and the circumstances chose me instead of the other way around! Barry McCoy taught a class on statistical mechanics and the Rogers–Ramanujan identities which got me hooked and I started working with him.

Mansour: What was the problem you worked on in your thesis?

Schilling: Actually, my thesis was not just on one problem, but consisted of about 8 papers I had written as a graduate student. The overarching topic of these papers is generalizations of the Rogers–Ramanujan type identities motivated by statistical mechanics and conformal field theory. The papers used different proof techniques such as Bailey pairs and recursions using finitizations of q-series identities.

I was planning to graduate in June of 1997, but I got a job offer from the University of Am-

[2] See `https://drive.google.com/file/d/1rq9OqnGv-Y3SXCrOOmbkBlZb8A-dhkSp/view`.

[3] R. Kedem, T. Klassen, B. McCoy, and E. Melzer, *Fermionic sum representations for conformal field theory characters*, Phys. Lett. B 307 (1993), 68.

sterdam that required me to start on April 1 (not an April's fools joke!). So I had to rush to get my thesis written.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Schilling: I would say it is neither, but rather a curiosity to understand why something is true. It might start with a specific problem and in the end, lead to a more theoretical approach to answer or view the problem.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Schilling: Sometimes the approach I use to solve a problem is what I would call "experimental mathematics". It involves discovering structures by doing examples or programming the objects on the computer and analyzing the data. When doing this, yes, often I feel something is true even before having proof. In fact, some properties might emerge from this method that helps to come up with proof.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Schilling: The development of the theory of Macdonald polynomials[4] has had a major impact on combinatorics and other fields. Many people have worked on this and this list includes a lot of important results such as the n! theorem by Haiman[5,6], the Shuffle Theorem by Carlsson and Mellit[7], the development of k-Schur functions by Lapointe, Lascoux, Morse and collaborators[8]. The theory of cluster algebra by Fomin and Zelevinsky[9] has been influential. The invention of quantum groups and subsequently the development of the theory of crystal bases by Kashiwara[10] have been extremely influential.

Well, I guess I cannot count to three since I have given you more than three results (depending on how you count!).

Mansour: What are the top three open questions in your list?

Schilling: It would be great to find combinatorial interpretations for various structure coefficients, such as the Kronecker coefficients[11], plethysm coefficients[12], Schubert and k-Schur function structure coefficients[13]. Finding a crystal structure on tableaux of tableaux[12,14]. Finding a symmetric chain decomposition of the Young subposet of partitions in a box[15].

Again, I am sorry, I have trouble counting to three!

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

Schilling: I would like to see answers to the above questions. It would also be great to see more interactions between combinatorics and other fields such as semigroup theory and probability. I am also wondering whether machine learning can help to find answers to some of the questions that are hard to solve right now.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Schilling: I prefer not to make a judgment about this. Science as a whole is important and mathematics can help us understand questions. I think it is best to keep an open mind on what will become important topics in the future.

Mansour: What do you think about the distinction between pure and applied mathemat-

[4] A. Garsia and J. B. Remmel, *Breakthroughs in the theory of Macdonald polynomials*, PNAS, 102(11) (2005), 3891–3894.

[5] M. Haiman, *Hilbert schemes, polygraphs, and the Macdonald positivity conjecture*, J. Amer. Math. Soc. 14(4) (2001), 941–1006.

[6] M. Haiman, *Vanishing theorems and character formulas for the Hilbert scheme of points in the plane*, Invent. Math. 149 (2002), 371–407.

[7] E. Carlsson and A. Mellit, *A proof of the shuffle conjecture*, J. Amer. Math. Soc. 31 (2018), 661–697.

[8] T. Lam, L. Lapointe, J. Morse, A. Schilling, M. Shimozono, and M. Zabrocki, *k-Schur functions and affine Schubert calculus*, Fields Institute Monographs, 33. Springer, New York; Fields Institute for Research in Mathematical Sciences, Toronto, ON, 2014.

[9] S. Fomin and A. Zelevinsky, *Cluster algebras I: Foundations*, J. Am. Math. Soc. 15 (2002), 497–529.

[10] M. Kashiwara, *Crystal bases and categorifications—Chern Medal lecture*, Proceedings of the International Congress of Mathematicians—Rio de Janeiro 2018. Vol. I. Plenary lectures, 249–258, World Sci. Publ., Hackensack, NJ, 2018.

[11] I. Pak and G. Panova, *Breaking down the reduced Kronecker coefficients*, C. R. Math. Acad. Sci. Paris 358:4 (2020), 463–468.

[12] L. Colmenarejo, R. Orellana, F. Saliola, A. Schilling, and M. Zabrocki, *The mystery of plethysm coefficients*, see `https://arxiv.org/abs/2208.07258`.

[13] R. P. Stanley, *Positivity problems and conjectures in algebraic combinatorics. Mathematics: frontiers and perspectives*, 295–319, Amer. Math. Soc., Providence, RI, 2000.

[14] N. A. Loehr and G. S. Warrington, *Quasisymmetric expansions of Schur-function plethysms*, Proc. Amer. Math. Soc. 140(4) (2012), 1159–1171.

[15] R. P. Stanley, *Weyl groups, the hard Lefschetz theorem, and the Sperner property*, SIAM J. Algebraic Discrete Methods 1(2) (1980), 168–184.

ics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called “pure” and “applied” mathematics?

Schilling: To me, it is not so important to focus on the question to distinguish pure and applied mathematics. The main difference might be that of motivation. If someone is driven by real-life problems, this person might be called an applied mathematician. On the other hand, the distinction could also come from the methods used to solve the problems. Applied mathematicians often work with models and sometimes solve them using numerical methods. Pure mathematicians do not always care about motivation, but study structures based on their own merit.

Mansour: What advice would you give young people thinking about pursuing a research career in mathematics?

Schilling: You cannot choose your parents, but you can choose your advisor! It is important to choose an advisor you can work with well, who shares your interests and supports you on the way. It is inevitable that there will be ups and downs when doing research, so it is important that you really like the problems you are working on. Also, do not be afraid to change course. It is not a failure not to be able to solve the problem you originally set out to solve. You might encounter other interesting questions on the way!

Mansour: It is an unpleasant fact that not many women follow postgraduate studies in mathematics. This prompts discussions about how to involve more women to research in mathematics. What do you think about this issue? What should be done in the next ten years to involve more women in mathematics?

Schilling: Providing an encouraging environment at all levels is very important! Combinatorics seems to attract more women than other areas in mathematics which I think is partially due to the fact that the community is in general very friendly and supportive. I must admit that I was quite skeptical at first about conferences targeted at women. At the first combinatorics conference I attended in Banff, I was surprised that the dynamic between women is very different when men are not present. It was a lot more open and collaborative. So events targeted directly at women and minorities will likely have a positive effect.

Mansour: Would you tell us about your interests besides mathematics?

Schilling: I like to play music. With some of my former postdocs and friends, we had a band called Tab Completion and wrote our own songs. I also love animals. With my dog Dobby I do agility (where he has to go over obstacles and jumps), some tricks, and play frisbee. With my horse Betsy Ross (she was born on July 4th) I train in dressage and also do a little bit of jumping. We have competed at local shows. I go hiking, bike riding, and swimming. At FPSAC I often organize frisbee games. And when the opportunity arises, I enjoy board or card games with friends, especially Hanabi.

Mansour: You are a frequent speaker at numerous conferences, workshops, and seminars. How important are such activities in your research career?

Schilling: Talks at conferences, workshops, and seminars are a great way to advertise my work. I have also often gotten helpful feedback on results or open questions I have talked about. Getting to talk to and interact with other mathematicians at these events is invaluable. During COVID, this did not happen for quite a while. My first in-person conference after COVID was OPAC in Minnesota. This was an extremely stimulating conference for me. I got so many great ideas talking to people there, listening to other talks, and getting feedback on my own talk. I realized how much I missed these stimulating interactions!

Mansour: From your web page, we see that you have some interest in puzzles. Do you enjoy solving them? What is your favorite one? Do they sometimes inspire you with an interesting math/combinatorics problem?

Schilling: Yes, I enjoy solving puzzles! I especially enjoyed the puzzles developed by Vivien Ripoll[16] for the online conference FPSAC 2020. My graduate students and I met at a coffee shop outdoors and solved them together there. That was fun in the midst of the pandemic. My favorite puzzle from the list on my website is the one about the painting. How can you hang up the painting with a string attached to two nails such that it will fall down if either of

[16] See `https://sites.google.com/view/fpsac2020online/events?pli=1`.

the two nails is removed? There are geometric and algebraic solutions to this puzzle.

Mansour: You have advised several Ph.D. students for their thesis. How important is it to collaborate with Ph.D. students and pass on knowledge to them? Do you maintain contact with your students?

Schilling: I have collaborated with almost all my students directly. For me, the easiest way to get them involved in research, pass on knowledge and how to approach research, and how write it up is by doing it together. At first, it might be a little one-sided, but after a while, they start teaching me new things they have discovered or read about. It is so satisfying seeing them develop into peers and make me into their student. I maintain contact with most of my former students.

Mansour: Recently, we have seen some unusual events in the math community. The editorial boards of some journals resigned and founded similar journals. For example, the Journal of Algebraic Combinatorics editors launched Algebraic Combinatorics. Same with the editors of the Journal of Combinatorial Theory, Series A, who founded Combinatorial Theory. In both newly established journals, you serve as a member of the editorial board. What is your opinion about this? Do you think this trend should continue with other journals as well? Do you think these projects will be long-term?

Schilling: Yes, I believe this trend should continue. I was very frustrated when I did a lot of editorial work for Elsevier journals and then did not have access to them any longer when Elsevier cut their contract with the University of California. This situation seemed absurd. So, when Vic Reiner voiced similar frustrations to the editorial board of JCTA, I joined him. We set up meetings with UC's eScholarship team who eventually provided a home for the new journal *Combinatorial Theory* and also set us up with a good funding model. The point of this trend is that the new journals are owned by mathematicians and are open-access. You can read more about this[17]. In fact, in addition to the two journals that you mention, I am joining the editorial team of Annals of Representation Theory which is another new open-access journal. We have secured solid funding for the projects, so I am confident that these projects are long-term. I hope that ECA will also have a big impact as one of the new open access journals.

Mansour: A few years ago, together with Daniel Bump, you published a book, *Crystal Bases: Representations and Combinatorics*[18]. What are crystal bases, and how are they related to combinatorics?

Schilling: Crystal bases are purely combinatorial objects that mirror representations of Lie algebras. They appeared in the work of Kashiwara, Lusztig, and Littelmann on quantum groups and the geometry of flag varieties[18]. Crystal bases arise in many unexpected places, from mathematical physics to probability and number theory. The book with Dan Bump provides an expository treatment of crystal bases from a purely combinatorial viewpoint. Unlike the original publications and expositions, where crystal bases were introduced as certain limits of representations of quantum groups when the quantum parameter q tends to zero, this book builds crystals through local axioms (based on ideas of Stembridge) and virtual crystals. The link between combinatorics and the representation theory is achieved using Demazure crystals and their characters.

Mansour: Some of your research deals with *Affine Schubert Calculus*[19]. How do combinatorics, in general, and enumeration, in particular, interact with Affine Schubert Calculus?

Schilling: Classical Schubert calculus is a branch of enumerative algebraic geometry concerned with questions such as how many lines in 3-space intersect four fixed lines. In general, lines are replaced by affine linear subspaces, and conditions on the dimensions of intersections are imposed. Using cohomology theory, these questions were translated into questions about symmetric functions. In particular, the structure coefficients of the Grassmannian $Gr(n, k)$ of k-planes in n-space turn out to be related to the Littlewood–Richardson coefficients $c^{\nu}_{\lambda,\mu}$, where λ, μ, ν are partitions. The

[17] `https://osc.universityofcalifornia.edu/2020/12/combinatorial-theory-launches/`.

[18] D. Bump and A. Schilling, *Crystal bases*, Representations and combinatorics. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017.

[19] For example, see J. Morse and A. Schilling, *Crystal approach to affine Schubert calculus*, Int. Math. Res. Not. IMRN 8, (2016), 2239–2294.

Littlewood–Richardson coefficients appear as structure coefficients of the Schur functions s_λ in $s_\lambda s_\mu = \sum_\nu c^\nu_{\lambda,\mu} s_\nu$. Hence the rich combinatorial backbone of the theory of Schur functions, including the Robinson–Schensted algorithm, jeu-de-taquin, the plactic monoid, and crystal theory, help to analyze Schubert calculus.

Affine Schubert calculus is the generalization when the Grassmannian is replaced by infinite-dimensional spaces known as affine Grassmannians. The cohomology can again be related to symmetric functions, in particular k-Schur functions. The combinatorics associated with these symmetric functions is very interesting and related to $(k+1)$-cores, k-bounded partitions, and the affine symmetric group.

Mansour: In a very recent joint work with Laura Colmenarejo, Rosa Orellana, Franco Saliola, and Mike Zabrocki, *The mystery of plethysm coefficients*[12], you introduced a new proof technique of combinatorial representation theory, which you called the "s-perp trick" and used it to give algorithms for computing monomial and Schur expansions of symmetric functions. Would you tell us about these results and the important ideas behind them? Did you uncover the entire mystery of plethysm coefficients, or is still there are works to be done?

Schilling: Actually, other people before us have used this trick. But we gave it a name and applied it to the setting of plethysm coefficients. The s-perp operator $s^\perp_\lambda$ is adjoint to multiplication by s_λ under the Hall inner product on symmetric functions, under which the Schur functions are orthonormal. The plethysm we consider is the character of the composition of two polynomial representations, say $\rho^\lambda : GL_n \to GL_m$ and $\rho^\mu : GL_m \to GL_\ell$, which is also a polynomial representation of GL_n. Its character is denoted by $s_\lambda[s_\mu]$. This operation can be viewed as an operation on symmetric polynomials, which was named (outer) plethysm by Littlewood. It is an open problem to find a combinatorial interpretation of the coefficients $a^\nu_{\lambda,\mu} \in \mathbb{N}$ in the expansion

$$s_\lambda[s_\mu] = \sum_\nu a^\nu_{\lambda,\mu} s_\nu.$$

The s-perp trick states that two symmetric functions f and g of homogeneous degree d are equal if $s^\perp_r f = s^\perp_r g$ for all $1 \leqslant r \leqslant d$. The cool observation we made is that there are explicit formulas for $s^\perp_r s_\lambda[s_\mu]$, which make it possible to compute the plethysm recursively. For certain cases (when λ is a certain hook and $\mu = (1^2)$ or (2)) we were able to use the recursions to find explicit combinatorial formulas for the plethysm coefficients. The problem is by no means solved in general, so the mystery remains, but the s-perp trick provides a computationally quicker way to compute the plethysm coefficients in some cases which was an unexpected benefit.

Mansour: In 2019, you and John Rhodes published a seminal paper titled *Unified theory for finite Markov chains*[20]. You used ideas from semigroup theory in the context of probability to develop this new theory that made it possible to compute the stationary distribution for any irreducible finite Markov chain. Would you tell us about the main ideas behind this research, some important applications, and possible future directions?

Schilling: One of the surprising and exciting new directions in my recent research is the application of the representation theory of semigroups to the study of Markov chains. In the seminal work of Bidigare, Hanlon, and Rockmore[21], which was continued by Brown, Diaconis, Athanasiadis, Björner, Chung, and Graham, amongst others, the special family of semigroups, now known as left regular bands first studied by Schützenberger in the forties, was applied to random walks or Markov chains on hyperplane arrangements. In his 1998 ICM lecture, Diaconis[22] discussed these developments. In Section 4.1, entitled *What is the ultimate generalization?* he asks how far the semigroup techniques can be taken.

In the article you mention above, John Rhodes and I provide a unified framework to compute the stationary distribution of any fi-

[20] J. Rhodes and A. Schilling, *Unified theory for finite Markov chains*, Adv. Math. 347 (2019), 739–779.

[21] P. Bidigare, P. Hanlon, and D. Rockmore, *A combinatorial description of the spectrum for the Tsetlin library and its generalization to hyperplane arrangements*, Duke Math. J. 99(1) (1999), 135–174.

[22] P. Diaconis, *From shuffling cards to walking around the building: an introduction to modern Markov chain theory*, In Proceedings of the International Congress of Mathematicians, Vol. I (Berlin, 1998), number Extra Vol. I, 187–204, 1998.

nite irreducible Markov chain or equivalently of any irreducible random walk on a finite semigroup S. Our methods use geometric finite semigroup theory via the Karnofsky–Rhodes and the McCammond expansions of finite semigroups with specified generators; this does not involve any linear algebra. The original Tsetlin library is obtained by applying the expansions to $P(n)$, the set of all subsets of an n element set. Our set-up generalizes previous groundbreaking work involving left-regular bands (or $\mathscr{R}$-trivial bands) by Brown and Diaconis[23], extensions to $\mathscr{R}$-trivial semigroups by Ayyer, Steinberg, Thiéry and myself[24], and important recent work by Chung and Graham[25]. The Karnofsky–Rhodes expansion of the right Cayley graph of S in terms of generators yields again a right Cayley graph. The McCammond expansion provides normal forms for elements in the expanded S. Using our previous results with Silva[26] based on work by Berstel, Perrin, and Reutenauer[27], we construct (infinite) semaphore codes on which we can define Markov chains. These semaphore codes can be lumped using geometric semigroup theory. Using normal forms and associated Kleene expressions, they yield formulas for the stationary distribution of the finite Markov chain of the expanded S and the original S.

One important question that remains to be answered in general is how to compute the mixing time of the Markov chain. In another recent paper[28], we gave upper bounds of the mixing time for a certain class of finite Markov chains by computing the expected length of paths to the minimal ideal in the Karnofsky–Rhodes/McCammond expansion of the right Cayley graph of the underlying semigroup. It would be great to make the bounds sharper and also to extend the results to a larger class of Markov chains.

Mansour: A common theme in several of your publications is *Kostka polynomials*. What kind of combinatorial problems involves them?

Schilling: Kostka polynomials[29,30] can be thought of as q-analogues of tensor product multiplicities or weight multiplicities in type A. They arose in my research when I was studying generalizations of the Rogers–Ramanujan type identities. There are formulas for them in terms of crystal bases with energy function statistics or in terms of rigged configurations with cocharge statistics. Rigged configurations are combinatorial objects which arose from the Bethe Ansatz in exactly solvable lattice models[31]. They can also be interpreted using Lascoux' and Schützenberger's charge on semistandard Young tableaux. They also appear when changing symmetric function bases in Macdonald theory. So as you can see, they are quite ubiquitous in combinatorics.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Schilling: My research is mostly in algebraic combinatorics. Sometimes I try to find bijections between sets, which give enumerative results. Or I try to find sets that count certain algebraically or geometrically defined numbers.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Schilling: I have lots of favorite results! However, there was one result with a spe-

[23] K. S. Brown and P. Diaconis, *Random walks and hyperplane arrangements*, Ann. Probab. 26(4) (1998), 1813–1854.

[24] A. Ayyer, A. Schilling, B. Steinberg, and N. M. Thiéry. *Markov chains, R-trivial monoids and representation theory*, Internat. J. Algebra Comput. 25(1-2) (2015), 169–231.

[25] F. Chung and R. Graham, *Edge flipping in graphs*, Adv. in Appl. Math. 48(1) (2012), 37–63.

[26] J. Rhodes, A. Schilling, and P. V. Silva, *Random walks on semaphore codes and delay de Bruijn semigroups*, Internat. J. Algebra Comput. 26(4) (2016), 635–673.

[27] J. Berstel, D. Perrin, and C. Reutenauer, *Codes and Automata*, Encyclopedia of Mathematics and Its Applications, vol. 129, Cambridge University Press, Cambridge, 2010.

[28] A. Schilling and J. Rhodes, *Upper bounds on mixing time of finite Markov chains*, SIAM J. on Discrete Math, to appear, arXiv:2010.08879.

[29] A. Lascoux and M. P. Schützenberger, *Sur une conjecture de H.O. Foulkes*, Comptes Rendus de l'Académie des Sciences, Série A-B. 286(7) (1978), A323–A324.

[30] A. Schilling and S. O. Warnaar, *Inhomogeneous lattice paths, generalized Kostka polynomials and A_{n-1} supernomials*, Comm. Math. Phys. 202:2 (1999), 359–401.

[31] A. N. Kirillov and N. Yu. Reshetikhin, *The Bethe Ansatz and the combinatorics of Young tableaux*, J. Soviet Math. 41 (1988), 925–955.

[32] A. Schilling, N. M. Thiéry, G. White, and N. Williams, *Braid moves in commutation classes of the symmetric group*, European J. Combin. 62 (2017), 15–34.

cial "eureka moment", which has appeared in the paper with Nicolas Thiéry, Graham White, and Nathan Williams[32] entitled "Braid moves in commutation classes of the symmetric group". This project started at a workshop at the American Institute of Mathematics (AIM). Nathan Williams and Vic Reiner stated the following conjecture at the workshop. The expected number of braid moves in the commutation class of the reduced word $(s_1 s_2 \cdots s_{n-1})(s_1 s_2 \cdots s_{n-2}) \cdots (s_1 s_2)(s_1)$ for the long element w_0 in the symmetric group S_n is one. Our group worked on the problem for the entire week at AIM. We wrote code to run experiments. When the week was over, Nicolas Thiéry and I took the AMTRAK train from San José to Davis. Nicolas was visiting me at the time. On the train along the beautiful bay, the idea for how to construct a bijection from the set $\mathsf{Red}(w_0)$ of reduced words for w_0 to the set of all braid moves in elements of $\mathsf{Red}(w_0)$ suddenly occurred to us. I guess our brains were saturated with ideas related to the problem for a full week and when the pressure of the intensity of the workshop lifted and the beautiful scenery unfolded, the final missing piece just presented itself. It was the best train ride of my life so far!

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Schilling: There are several problems I have been working on for many years. One of them is the plethysm problem mentioned above. By trying to understand it, my collaborators and I have come across many interesting structures such as the representation theory of the uniform block permutation algebra. So there is lots of progress even though it might only be tangential to the original problem.

Mansour: In a very recent short article, published in the newsletter of the European Mathematical Society, Melvyn B. Nathanson, while elaborating the ethical aspects of the question "Who Owns the Theorem?" concluded that "Mathematical truths exist, and mathematicians only discover them." On the other side, there are opinions that "mathematical truths are invented". As a third way, some people claim that it is both invented and discovered. What do you think about this old discussion?

Schilling: Without having read the article that you cited, I also believe mathematical truth exists and we are here to witness it, to make it our own by understanding it and presenting it in a way that others can grasp it. The same mathematical truth can be discovered by different people from different angles or through different methods. So I guess in this sense the people who really grasp the meaning of a theorem own it. The better it is written down and presented, the more people can own the theorem.

Mansour: Professor Anne Schilling, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.