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Interview with Joel Spencer

Toufik Mansour

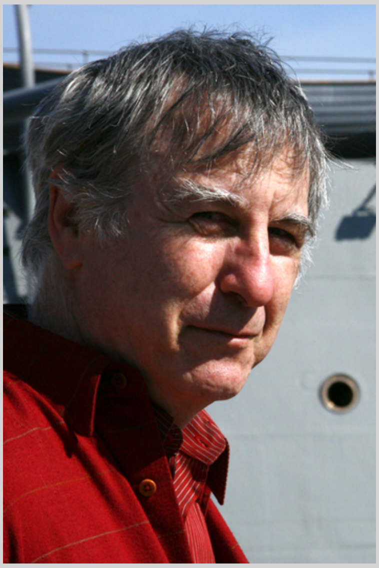

Photo by Mary Ann Spencer.
Backdrop is Aurora in St. Petersburg.

Joel Spencer completed his Ph.D. at Harvard University in 1970 under the supervision of Andrew Gleason. During his career he worked at UCLA (1971-1972), MIT (1972-1975), and Stony Brook University (1975-1988). Since 1988 he has been a professor at NYU's Courant Institute of Mathematical Sciences. He has held visiting positions in Budapest (as a N.A.S. Exchange Fellow), the Weizmann Institute, Reading, MIT, the Institute for Mathematics and Its Applications, the Institute for Advanced Study, Melbourne, and at the Mittag-Leffler Institute. He is a Co-Founder and Co-Editor-in-Chief of journal Random Structures and Algorithms (1990-2007) and also a member of the Editorial Board for the journals Combinatorica (1979-2009); American Math Monthly (1986-1991); Discrete Mathematics (1988-1996); The Annals of Applied Probability (1990-1994). He was an invited speaker at the International Congress of Mathematics in Zürich in 1994. In 2012 he was made a Fellow of the AMS and in 2017 became a SIAM Fellow. In 2021 he received the Leroy P. Steele Prize for mathematical exposition with his coauthor Noga Alon for their book The Probabilistic Method. His Erdős number is one.

Mansour: Professor Spencer, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Spencer: Combinatorics deals with discrete structures in many forms. On one side it is an enumeration. On another, it is designs, such as Steiner Triple Systems. But there is much much more and it is difficult to pinpoint.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Spencer: So very very exciting. Most prominently, theoretical computer science. The analysis of algorithms is essentially combinatorics. The proofs of Szemerédi's theorem using topological dynamics, Fourier analysis, and other means show the great unity of mathematics. Combinatorial number theory is yet another. I feel today that combinatorics, like linear algebra and calculus, is one of the bedrocks of mathematics that is used throughout our discipline.

Mansour: What have been some of the main goals of your research?

Spencer: Fifty years ago probabilistic methods were a collection of beautiful papers by Paul Erdős and a few others. I have strived to make probabilistic methods a cohesive theory. Among my proudest moments is going to math departments (including my alma mater MIT) and seeing probabilistic methods being given not as a special topics course but as a regular part of the graduate curriculum.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some

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other people?

Spencer: I had an inspirational high school teacher, Ira Ewen. He showed me how mathematics was an exciting living field. I recall him showing me the Twin Prime Conjecture. I went home and worked on it for several days [getting nowhere, of course] and then I had an epiphany: the Twin Prime Conjecture is either absolutely true or absolutely false and so mathematicians are searching for absolute truth.

Mansour: Were there specific problems that made you first interested in combinatorics?

Spencer: While in high school a pair of orthogonal Latin squares of order ten was found and this fascinated me. Years later I attended a series of lectures by Haim Hanani on block designs. I never actually accomplished much in this area but I loved working in it.

Mansour: What was the reason you chose Harvard University for your Ph.D. and your advisor Andrew Gleason?

Spencer: It was a big mistake! I simply chose Harvard for its name. While Harvard had (and has) great math at that time the areas (such as algebraic geometry) did not interest me and Harvard had little interest in combinatorics. Indeed, after two years, I left Harvard. I like to say (with slight exaggeration) that for me Harvard was a mail-order Ph.D. program. Gleason was the exception, a truly remarkable man with great insights in many many areas, including combinatorics.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Spencer: Specific problems, definitely. In that sense (among others) I am strongly in the Erdős camp. Tim Gowers has a remarkable paper, The two cultures of mathematics[1], comparing and contrasting the problem solvers and the theory builders. Highly recommended!

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Spencer: Usually, though not always. It helps to think the result is true–even when it turns out to be false it pushes me along.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Spencer: I will give four in probabilistic combinatorics: • Applying probabilistic methods in a novel way to prove the existence of exact designs, such as Steiner Triple Systems, initiated by Peter Keevash[2]. • Analysis of the random triangle-free process via differential equations, by Bohman[3], Morris[4,5] and others. • The continuing reproving and extending Szemerédi's Theorem, using many areas of mathematics, by so many people, most recently Peluse[6]. • The development of graphons, by Lovász, Chayes and many others[7,8,9] to deal with large (technically, asymptotic sequences) graphs.

Mansour: What are the top three open questions in your list?

Spencer: • The Komlós Conjecture[10]: given $\vec{v}_i \in R^d$ in the Euclidean ball some $\sum \epsilon_i \vec{v}_i$, all $\epsilon_i \in \{-1,+1\}$, has L^∞ norm at most some absolute (independent of d) constant C. • Close bounds on functions given by van der Waerden and Szemerédi's theorems. That is, if $\{1,\ldots,K^n\}$ (K large) is two colored must there exist a monochromatic arithmetic progression of length n. Gowers[11] had breakthroughs in these functions but the gap is still wide. • Erdős's question on the maximal number of unit distances from n points in the plane[12]. Many feel it should be $n^{1+o(1)}$.

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

[1] See `https://www.dpmms.cam.ac.uk/~wtg10/2cultures.pdf`.

[2] P. Keevash, *Counting Steiner triple systems*, Collected volume, European Congress of Mathematics, Berlin, Editors Volker Mehrmann and Martin Skutella, 2016, 459–481.

[3] T. Bohman, *The triangle-free process*, Adv. in Math. 221:5 (2009), 1653–1677.

[4] J. Balogh, R. Morris, and W. Samotij, *Random sum-free subsets of abelian groups*, Israel J. Math. 199(2) (2014), 651–685.

[5] G. F. Pontiveros, S. Griffiths, and R. Morris, *The triangle-free process and the Ramsey number $R(3,k)$*, Memoirs AMS, 1274.

[6] S. Peluse, *On the polynomial Szemerédi theorem in finite fields*, Duke Math. J. 168(5) (2019), 749—774.

[7] C. Borgs, J. Chayes, L. Lovász, V. T. S ós, B. Szegedy, and K. Vesztergombi, *Graph limits and parameter testing*, in Proceedings of the 38th Annual ACM Symposium on Theory of Computing, New York, 2006, 261–270.

[8] C. Borgs, J. T. Chayes, L. Lovász, V. T. S ós, and K. Vesztergombi, *Convergent sequences of dense graphs I: Subgraph frequencies, metric properties and testing*, Adv. Math. 219 (2008), 1801–1851.

[9] C. Borgs, J. T. Chayes, L. Lovász, V. T. S ós, and K. Vesztergombi, *Convergent sequences of dense graphs II. Multiway cuts and statistical physics*, Ann. Math. 176 (2012), 151–219.

[10] J. Spencer, *Ten Lectures on the Probabilistic Method*, Second Edition, SIAM, 1994.

[11] W.T. Gowers, *A new proof of Szemeredi's theorem*, GAFA, Geom. funct. anal. 11 (2001), 465–588.

[12] N. Alon, M. Bucić, and L. Sauermann, *Unit and distinct distances in typical norms*, arXiv:2302.09058v1.

Spencer: It has been a great joy to me that probabilistic methods have grown so much, that there are so many people applying deep methods to get sparkling new results. I am so happy to sit back and see yet more!

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Spencer: Calculus, linear algebra, combinatorics, number theory – these are core in the sense that all mathematicians should have some facility with them. While some results are more important than others I would not want to rank the areas in importance. When I was starting out Combinatorics was at the bottom of many lists! We have come a long way!

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Spencer: An important mentor for me, D. Ray Fulkerson (a key figure in the development of Operations Research) would say – there is no pure mathematics and applied mathematics, there is only good mathematics. These are words that I have lived by.

Mansour: You have supervised several students for their Ph.D. thesis. What do you think about the importance of working with Ph.D. students and passing knowledge to them? Do you follow your students after they complete their thesis?

Spencer: Extremely important both to the student and the advisor. I am so pleased that many of my advisees are now having successful careers and that we stay in touch. The passing on of specific knowledge is important but perhaps more so giving the student an idea of the mathematical world.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Spencer: Take the area that fits you best and work on it. At the same time (and this is difficult while working on a thesis), do not be too narrow – read and study from many areas of mathematics. Very important: Be scientifically gregarious – go to conferences, and talk to people about your work and theirs.

Mansour: You gave talks at numerous conferences, workshops, and seminars. What do you think about the importance of such activities for the progress of science and the scientific community?

Spencer: Absolutely essential. While I love Zoom, nothing takes the place of going to a conference. Talking with people over coffee is the springboard for mathematical progress.

Mansour: Your book, *The Probabilistic Method*[13], which you coauthored with Noga Alon, is a classic in the field. In 2021, together with Noga Alon, you received the *Leroy P. Steele Prize* for mathematical exposition. Please tell us about your perspective on writing a textbook or research monograph. What features of a successful book distinguish it from others?

Spencer: I have never written a textbook, though the probabilistic method has been used as one. I aimed to describe a methodology. I wanted beautiful proofs. I sacrificed the most complete results (though not the rigor of the proofs) in order to give the reader a better understanding. A key step is selecting the right coauthor, and Noga[14] is the best there is.

Mansour: Which results do you consider the most successful applications of the Probabilistic Method?

Spencer: I am not big on applications so I will give three of the most beautiful proofs. • Cherkashin and Kozik[15] on coloring vertices so as to avoid monochromatic n-sets. • Shamir on concentration of chromatic number for $G(n, p)$. While I coauthored the key paper[16], my contribution was minimal. All the honors to Eli! • Erdős's two page lower bound on $R(k, k)$[17].

Mansour: The title of one of your books is

[13] N. Alon and J. Spencer, *The Probabilistic Method*, J. Wiley and Sons, New York, NY, 2nd edition, 2000.

[14] T. Mansour, *Interview with Noga Alon*, Enumer. Combin. Appl. 1:1 (2021), Interview S3I2.

[15] D. D. Cherkashin and J. Kozik, *A note on random greedy coloring of uniform hypergraphs*, Random Struct. Algorithms 47:3 (2014), 407–413.

[16] E. Shamir and J. Spencer, *Sharp concentration of the chromatic number on random graphs $G_{n,p}$*, Combinatorica 7 (1987), 121–129.

[17] P. Erdős, *Some remarks on the theory of graphs*, Bull. Am. Math. Soc. 53 (1947), 292–294.

[18] J. Spencer, *The Strange Logic of Random Graphs*, Part of the book series: Algorithms and Combinatorics (AC, volume 22), 2001.

The Strange Logic of Random Graphs[18] You have also written several papers on *sparse random graphs*[19,20]. Why are they important and strange? Are they also interesting combinatorial objects for enumeration?

Spencer: As Erdős and Rényi showed eighty years ago, the random graph $G(n, p(n))$ evolves through various regimes. For example, K_4s appear when $p = \Theta(n^{-2/3})$. This monograph combined graphs, probability, and logic. It examined the behavior relative to any property written in (for the most part) first-order language. "Strange" was a colorful word to use – and does communicate how I see the subject.

Mansour: In a joint paper with Remco van der Hofstad, *Counting connected graphs asymptotically*[21], you found the asymptotic number of connected graphs with k vertices and $k - 1 + \ell$ edges when k, ℓ tend to infinity, thus reproving a result of Bender, Canfield, and McKay. Please explain the main ideas behind this result and the methods used.

Spencer: This is a favorite paper of mine as it combined combinatorics, probability, and algorithms. With massaging we want the probability that $G(k, p)$ is connected with $k - 1 + \ell$ edges. Designate an initial vertex and apply Breadth-First Search. With p cleverly chosen and appropriate conditioning the Queue Size $Q(t)$ acts asymptotically like a Brownian motion, with decay as t increases. We find the probability that the Bridge never goes negative – so that BFS finds all the vertices and G is connected. I loved the combination of methods and Remco was a master of them all, plus his invaluable background in mathematical physics.

Mansour: Another research interest of yours is Ramsey theory. In your book with Graham and Rothschild[22], you write "Ramsey theory has only been recognized within the last ten years as a cohesive subdiscipline of combinatorial analysis. The basic philosophy underlying the theory is that some regularity must always exist within any sufficiently large system. To quote the late T. S. Motzkin: 'Complete disorder is impossible.' Consequently, it is not surprising that results from Ramsey theory occur throughout a large part of mathematics.—Ramsey theory is a jewel of pure mathematics." Would you elaborate on these thoughts more? Did we witness some spectacular results in this theory during the last ten years? Can you list some long-standing open problems you are curious about?

Spencer: Some areas, like Galois Theory and Ramsey Theory, are intensely beautiful, while others are more quotidian. It is mysterious how this happens. Personally, I have been attracted to Ramsey theory since high school when my teacher asked me to show $R(3, 3) = 6$. I did come up with a twenty-page case-by-case argument. When I saw the book proof I was hooked.

The asymptotics of $R(k, k)$ is, for me, a central problem in both Ramsey theory and the probabilistic method. The bounds (roughly) of $\sqrt{2}^k$ and 4^k have been known since I was born. There have been improvements on subexponential terms but this gap remains stubbornly elusive.

Mansour: Paul Erdös was one of your collaborators. Can you tell us about his approach to mathematical problems? How was he as a person?

Spencer: Uncle Paul was, is, and forever will be the center of my mathematical universe. This is true for me and for so many other mathematicians. Paul had a special spirit – he was a searcher for mathematical truth. Mathematicians need faith, that their abstract thoughts have a Platonic reality. Paul embodied that faith. And, as religious people know, to witness faith is to be given faith. At least, that is what happened to me. Paul did not create mathematical talent. But working with Paul brought me and countless others to a new level.

Paul's approach was to look at specific problems. In retrospect, work on these problems led to the development of large theories, including the Probabilistic Method. Paul had a deep insight, the problems he chose were anything but random – to solve one of his problems (and, perhaps, win a 20-dollar prize!) one need a new approach, one needed to extend the borders of our knowledge.

Mansour: The Bohman-Frieze process is

[19] L. Babai, M. Simonovits, and J. Spencer, it Extremal subgraphs of random graphs, J. Graph Theory 14(5) (1990), 599–622.
[20] S. Shelah and J. Spencer, *Zero-one laws for sparse random graphs*, J. Amer. Math. Soc. 1:1 (1988), 97–115.
[21] R. van der Hofstad and J. Spencer, *Counting connected graphs asymptotically*, European J. Combin. 27:8 (2006), 1294–1320.
[22] R. L. Graham, B. L. Rothschild, and J. Spencer, *Ramsey Theory*, John Wiley & Sons, 1991.

another interesting random graph process[23]. What are the main characteristics of this process, and how different is it from the Erdös-Rényi process?

Spencer: Bohman-Frieze was a conceptual breakthrough, opening the door to random graph processes other than Erdős-Rényi. Roughly, the added edge is chosen by a random process, but one that gives more weight to edges that join two isolated vertices. Erdős-Rényi has percolation (the creation of a giant component) at $tn/2$ edges with $t = 1$. With Nick Wormald[24] we were able to find a differential equation $y(t)$ simulating Bohman-Frieze with $y(t) \to \infty$ at some $t = t_0$. We could then show that the giant component was created at that time. Much further work by Riordan, Warnke[25] and many others show that at percolation Bohman-Frieze behaves similarly to Erdős-Rényi.

Particularly intriguing has been the "product" rule in which, roughly, an edge $\{v, w\}$ is more likely to be added when the product of the sizes of the components containing v, w respective is small. Data shows a very sharp percolation, though it is known[26] (suitably normalized) to be continuous. The nature of the percolation – is most certainly different from Erdős-Rényi – remains unknown and simulation, even with 10^9 vertices, has proven to be deceptive.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Spencer: Enumeration – for me asymptotic enumeration – is the "addition" of combinatorics, and is absolutely central.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Spencer: Randomly coloring n sets on n vertices gives a discrepancy of $c\sqrt{n \ln n}$. Erdős asked if one can do better. I looked at this problem, off and on, for years. I was at a conference in Vienna and the lecture was boring. I started looking yet again at the problem. Yes, it was definitely a eureka moment. By the end of the lecture, I stood up and told my friend Miki Simonovits that I had the solution, that there existed a coloring with discrepancy $O(\sqrt{n})$. That is, there exists a Red/Blue coloring so that all sets have a discrepancy less than $6\sqrt{n}$. (This led to the catchy title – Six Standard Deviations Suffice – though since then the six has been reduced.) It took six months of careful writing and various extensions but I had been right, I had the argument in that hour. It has only happened once in my life. But once is enough!

I am pleased that the result remains vital. There are now many proofs, far better than mine. The biggest breakthrough was by Bansal[27] who gave an efficient algorithm for finding the coloring, and now there are several algorithms. My personal favorite is by Lovett and Meka[28] who use floating colors [starting at 0 and floating to ± 1] with the color vector moving in a carefully restricted Brownian motion.

Mansour: Is there a specific problem you have worked on for many years? What progress have you made?

Spencer: Erdős showed in 1963 that given any $k2^{n-1}$ sets of size n with $k < 1$ there is a Red/Blue coloring of the underlying vertices so that no set is monochromatic. Basically, he showed that a random coloring would work. What about larger k? In 1964 he showed that for $k = cn^2$ this does not hold. Raising k above 1 has been worked on by many people. Beck had a major breakthrough, getting k roughly $n^{1/3}$. Srinivasan, Radhukrishnan[29], Kozik and Cherkashin[15] showed it was true for k roughly $\sqrt{n}$. I've worked intensely, albeit on and off, on this problem since 1968. What progress have I made? None!

[23] M. Kang, W. Perkins, and J. Spencer, *The Bohman-Frieze process near criticality*, Random Struct. Algorithms 43:2 (2013), 221–250.

[24] J. Spencer and N. Wormald, *Birth control for giants*, Combinatorica 27:5 (2008), 587–628.

[25] O. Riordan and L. Warnke, *The phase transition in bounded-size Achliopta processes*, arXiv: 1704.08714.

[26] O. Riordan and L. Warnke, *Achlioptas process phase transitions are continuous*, Ann. Appl. Prob. 22 (2012), 1450–1464.

[27] N. Bansal, *Constructive algorithms for discrepancy minimization*, in Foundations of Computer Science (FOCS) (2010), 3–10.

[28] S. Lovett and R. Meka, *Constructive discrepancy minimization by walking on the edges*, in Foundations of Computer Science (FOCS) (2012), 61–67.

[29] J. Radhakrishnan and A. Srinivasan, *Improved bounds and algorithms for hypergraph 2-coloring*, Random Struct. Algorithms 16(1) (2000), 4–32.

Lord Kelvin said it best: “One word characterizes the most strenuous of the efforts for the advancement of science that I have made perseveringly during 55 years. That word is failure.”

Mansour: In a very recent short article published in the Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question “Who Owns the Theorem?” concluded that “Mathematical truths exist, and mathematicians only discover them.” Conversely, there are opinions that “mathematical truths are invented.” As a third way, some claim it is both invented and discovered. What do you think about this old discussion?

Spencer: I am absolutely in the Erdős camp on this one. A cornerstone of his theology was The Book, containing all mathematical theorems. We do not create mathematics, we read from the pages of The Book. Erdős read chapters, perhaps I have read a few lines.

I should add, however, that the mathematical problems we choose to study are very much influenced by the world around us. My favorite example is $P = NP$? – what Smale called Computer Science’s gift to Mathematics. If we didn’t have computers the mathematical study of algorithms, as pioneered with Knuth, might never have come into being.

Mansour: Professor Spencer, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

Spencer: It is been my pleasure!