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Interview with Douglas West

Toufik Mansour

Photo by University of Illinois Urbana-Champaign, College of Liberal Arts & Sciences

Douglas B. West completed his undergraduate studies at Princeton University in 1974. He obtained his Ph.D. in 1978 at the Massachusetts Institute of Technology under the supervision of Daniel J. Kleitman. During his career, he taught and conducted research at Stanford University (1978–79), Princeton University (1979–82), the University of California, Berkeley (1989–90), the University of Illinois (1982–present), and Zhejiang Normal University (2012–present). Since 2011 he has been a Professor Emeritus at the University of Illinois.
Professor West has been the Editor-in-Chief of the journal Discrete Mathematics since 2017 and has advised 38 Ph.D. theses. He has hosted an up-to-date listing of conferences in discrete mathematics on his homepage since 1997 and is known for his textbooks *Introduction to Graph Theory* (Prentice-Hall 1996, 2001) and *Combinatorial Mathematics* (Cambridge 2021).

Mansour: Professor West, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?
West: Thank you for the honor. I think of combinatorics as the study of finite sets and the arrangements and structures that are placed upon them. That provides the context, but equally important to the flavor of the subject are the types of questions that are asked. I think these are of three types: existence, enumeration, and extremality.
Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?
West: This has been a very important development in the subject, leading to significant advances. Techniques from other areas have influenced combinatorics, and combinatorial results have found applications in other areas. Probabilistic[1], algebraic[2], and topological[3] techniques have led to solutions of combinatorial problems. Ramsey theory[4], the Regularity Lemma[5], and the Combinatorial Nullstellensatz[6] have helped to explain large structures and yield applications in number theory and other areas.
Mansour: What have been some of the main goals of your research?
West: My research was never driven by a particular goal. I tried to formulate and/or solve problems that my students, colleagues, and I

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[1] N. Alon and G. H. Spencer, *The probabilistic method*, Fourth edition, Wiley Series in Discrete Mathematics and Optimization, John Wiley & Sons, Inc., Hoboken, NJ, 2016.
[2] L. Babai and P. Frankl, *Linear Algebra Methods in Combinatorics*, Univ. Chicago, 1992. See `https://people.cs.uchicago.edu/~laci/babai-frankl-book2022.pdf`.
[3] J. Matoušek, *Using the Borsuk-Ulam theorem*, Lectures on topological methods in combinatorics and geometry. Written in cooperation with Anders Björner and Günter M. Ziegler, Universitext, Springer-Verlag, Berlin, 2003.
[4] R. L. Graham, B. L. Rothschild, and J. H. Spencer, *Ramsey theory*, Paperback edition of the second (1990) edition, Wiley Series in Discrete Mathematics and Optimization, John Wiley & Sons, Inc., Hoboken, NJ, 2013.
[5] E. Szemerédi, *Regular partitions of graphs, Problémes combinatoires et théorie des graphes* (Colloq. Internat. CNRS Univ. Orsay, Orsay, 1976), vol. 260, CNRS, 1978, 399–401.
[6] N. Alon, *Combinatorial Nullstellensatz*, Combin. Probab. Comput. 8(1-2) (1999), 7–29.

found interesting. I am reminded of the interview[7] of John Conway in *Math Horizons* in which he exclaimed about how marvelous it is that one can get paid to have so much fun. Nevertheless, one theme in my work has been the aim of generalizing appealing examples or results to larger classes of structures.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

West: As a child I liked to solve puzzles and play games; I assembled many jigsaw puzzles and played a lot of board games. Instead of doodling, I drew road networks; the lanes were 1/4 inch wide. A friend of my father gave me a couple of books on recreational mathematics including one by Martin Gardner[8] as a birthday present; I quite enjoyed them. My parents did not push me in a particular direction, but they were not surprised when I wound up in mathematics. In eighth grade, my mathematics teacher refused to answer my questions in class but established a two-minute period after the class ended in which I could raise my objections to what he had said during the class.

Mansour: Were there specific problems that made you first interested in combinatorics?

West: My first formal contact with combinatorics was a seminar course in my first year as an undergraduate at Princeton. It was offered by Tom Tucker in 1972, based on the early *Introduction to Combinatorial Mathematics* by C.L. Liu[9], published in 1968. This text established a broad-based introduction to the field at an undergraduate level. (Ten years later I wound up at the University of Illinois, where C.L. Liu was teaching in the Computer Science Department, and that led me to writing books[10,11].) What intrigued me was the flavor of the arguments rather than specific problems.

Mansour: What was the reason you chose the Massachusetts Institute of Technology for your Ph.D. and your advisor Daniel J. Kleitman?

West: After my undergraduate studies, I had no idea what I wanted to do with my life. What I did know was that if I was going to do mathematics, then what I would do in mathematics would be combinatorics. So, I asked Albert Tucker at Princeton where one should go to do graduate work in combinatorics, and he said the Massachusetts Institute of Technology (MIT). So, I applied to MIT and only to MIT, reasoning that if I did not get in, then I would have to take time off and figure out what I wanted to do. For that reason I felt a bit disappointed when I was given an NSF Graduate Fellowship, because if they were going to pay me to go to graduate school it seemed sensible to do so, even though it would delay figuring out what I wanted to do. Years later I realized that if I was not doing mathematics, then I would want to be doing something editorial, and I was lucky enough to have followed a path that would enable me to do both! So, I started turning course notes into books and lucked into editorial positions for the problems section of the *American Mathematical Monthly* and later *Discrete Mathematics* and lived happily ever after. I consider myself to have been very lucky in my career.

At MIT at the time were Dan Kleitman, Gian-Carlo Rota, and a young Richard Stanley[12], all quite brilliant. Rota and Stanley were theory-builders whose work expanded on large amounts of sophisticated mathematics; Kleitman was a problem-solver. For someone in my haphazard and uneducated state, the choice of advisor was obvious.

Mansour: What was the problem you worked on in your thesis?

West: I worked on my undergraduate thesis for 28 days before it was due and 30 days after it was due: that was a program written in SNOBOL to play backgammon. My doctoral thesis discussed three problems, though I do not recall which three. One may have been an extremal problem about the number of meets between elements of two subfamilies of a distributive lattice[13], which grew out of a paper Kleitman asked me to present in a

[7] `https://www.ams.org/notices/201305/rnoti-p567.pdf`.

[8] M. Gardner, *The Scientific American Book of Mathematical Puzzles and Diversions*, Simon and Schuster, 1959.

[9] C. L. Liu, *Introduction to combinatorial mathematics*, McGraw-Hill Book Co., New York-Toronto-London, 1968.

[10] D. B. West, *Introduction to graph theory*, Prentice Hall, Inc., Upper Saddle River, NJ, 1996.

[11] D. B. West, *Combinatorial mathematics*, Cambridge University Press, Cambridge, 2021.

[12] Interview with Richard P. Stanley, ECA 1:1 (2021) Interview S3I1.

[13] D. E. Daykin, D. J. Kleitman, and D. B. West, *The number of meets between two subsets of a lattice*, J. Combin. Theory Ser. A 26:2 (1979), 135–156.

seminar. Another extremal problem concerned semiantichains in products of skew chain orders[14,15], which is the reason I am part of the mysterious G. W. Peck[16]. I spent a summer trying to improve the result by Bill Gates and Christos Papadimitriou on the Pancake Problem[17], but I was never able to finish all the cases to get a result. On the other hand, another summer of fiddling did produce an inductive construction of a symmetric chain decomposition of Stanley's $L(4, n)$ poset[18]; this was an existence question. Finally, I spent a lot of time studying a special class of solutions to the Gossip Problem[19], which involved mostly enumerative questions.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

West: The general theoretical questions that I have studied are just more general phrasings of specific problems. One can move up and down the scale of generality, trying to generalize specific problems that have been solved or to restrict unsolved general questions to more specific classes where they are more tractable.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

West: Yes, generally we work with strong feelings about what the answer is. Sometimes we are wrong. Hopefully we discover our mistakes by explaining the proof to colleagues or students before we publish the papers (not always, unfortunately). In the age of TEX and the decline of memory, in recent decades I also start writing the paper before finishing the proof.

On the other hand, sometimes different participants have different opinions about what is true. I remember working on several problems with Tom Trotter where I thought the answer was bounded and he thought it was unbounded. Sometimes I was right; sometimes he was right.

Mansour: What three results do you consider the most influential in graph theory during the last thirty years?

West: Since I am a problem-solver rather than a theory-builder, I am not a good person to answer this question. Nevertheless, there are some obvious examples with far-reaching implications. I will not confine myself to 30 years, and I will not mention anything terribly recent, due to my limited knowledge and limited vision about the consequences of recent results.

The Graph Minors Project of Robertson and Seymour ushered in a new way of studying the structure of graphs. Szemerédi's Regularity Lemma and its subsequent enhancements made it possible to asymptotically prove many statements for sufficiently large graphs that still seem hopelessly complicated to prove for all graphs. The probabilistic method is a collection of tools originating almost 75 years ago, but its development over the past 30 or 40 years has led to many advances and simplifications; combinatorics initially studied very special structures, but now we have the ability to study "typical" structures and make such notions precise.

Mansour: Is there a result in graph theory you consider extremely surprising or unexpected?

West: I think the gap between choosability and paintability for the complete bipartite graph $K_{n,n}$ was unexpected. The *choosability* (or *choice number* or *list chromatic number*) of a graph (introduced by Vizing[20] in 1976) is the least k such that a proper coloring can be chosen from lists of colors available at the vertices whenever all the lists have size k[21]. This is at least the chromatic number, since the lists could be identical. The *paintability* (introduced by Schauz[22] and by Zhu[23] in 2009) is given by an on-line version of list coloring

[14] G. W. Peck, *Maximum antichains of rectangular arrays*, J. Combin. Theory Ser. A 27:3 (1979), 397–400.

[15] D. B. West and D. J. Kleitman, *Skew chain orders and sets of rectangles*, Discrete Math. 27:1 (1979), 99–102.

[16] For the story of G. W. Peck, see G. W. Peck, *Kleitman and combinatorics: a celebration*, Discrete Math. 257:2–3 (2002), 193–224.

[17] W. H. Gates and C. H. Papadimitriou, *Bounds for sorting by prefix reversal*, Discrete Math. 27:1 (1979), 47–57.

[18] D. B. West, *A symmetric chain decomposition of $L(4, n)$*, European J. Combin. 1:4 (1980), 379–383.

[19] D. B. West, *A class of solutions to the gossip problem*, I. Discrete Math. 39:3 (1982), 307–326. II. Discrete Math. 40:1 (1982), 87–113. III. Discrete Math. 40:2-3 (1982), 285–310.

[20] V. G. Vizing, *Coloring the vertices of a graph in prescribed colors*, (Russian) Diskret. Analiz 1976, no. 29, Metody Diskret. Anal. v Teorii Kodov i Shem, 3–10, 101.

[21] For a survey of the major results on list coloring, see M. Stiebitz and M. Voigt, *List-colourings*, Topics in chromatic graph theory, 114–136, Encyclopedia Math. Appl. 156, Cambridge Univ. Press, Cambridge, 2015.

[22] U. Schauz, *Mr. Paint and Mrs. Correct*, Electron. J. Combin. 16:1 (2009), Paper 77.

[23] X. Zhu, *On-line list colouring of graphs*, Electron. J. Combin. 16:1 (2009), Paper 127.

and is always at least the choosability. The choosability of $K_{n,n}$ is known[24] since 2000 to be at most $\lg n - (\frac{1}{2} + o(1)) \lg \lg n$. Until Duraj, Gutowski, and Kozik[25] proved that the paintability of $K_{n,n}$ is at least $\lg n + O(1)$, no examples were known with paintability exceeding choosability by more than 1. Now the difference can be arbitrarily large.

Mansour: What are the top three open questions in your list?

West: My personal favorites are the Reconstruction Conjecture[26] of Kelly and Ulam for graphs (and Manvel's extension[27] of it to decks obtained by deleting ℓ vertices instead of one vertex), the existence of a symmetric chain decomposition of the poset $L(m, n)$[28], and the Pancake Problem[29] (the worst-case number of prefix reversals needed to sort a permutation of $[n]$).

Mansour: On your web page, you maintain a section called *Open Problems - Graph Theory and Combinatorics*[30]. Which is your favorite problem from that list?

West: That is a very old and out-of-date page where I started to gather well-known open problems, but my attention turned to other things, especially when Bojan Mohar and his colleagues at Simon Fraser developed the much more thorough Open Problems Garden[31]. Looking back at my page now, I see lots of very beautiful conjectures; it makes me want to go back and polish up the page with more open questions. If I have to pick a favorite, today it is the List Coloring Conjecture[32] stating that the edge-choosability of any graph equals its edge-chromatic number.

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

West: I would like to see brilliant combinatorial proofs of many long-standing conjectures.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

West: Yes, but there will be endless debate about what they are. When I was young and naive, I did not think it was important for discrete mathematicians to know much continuous mathematics, and presumably vice versa. Now I regret these gaps in my knowledge. After a basic "transition" course that introduces students to the fundamental ideas of mathematical reasoning, there should be courses that explain the most important parts in a variety of areas. I cannot say what the areas are, but the people in those areas can determine what should be known about them.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

West: I think of this less as a distinction in topics or techniques and more as a distinction in motivation. Applied mathematics is devoted to solving problems arising in the real world, which can be messy. Pure mathematics happens when we do not worry about applications and instead focus on the beauty or elegance of the question or the argument. To make progress, we need people who move between the two, extracting interesting pure questions from the applications and applying the beautiful theorems. I think the great divide is between discrete and continuous rather than between pure and applied, but even they interact more than they used to.

Mansour: You frequently provide good problems to the *American Mathematical Monthly* and also published a book, together with John D'Angelo, *Mathematical Thinking: Problem-*

[24] J. Radhakrishnan and A. Srinivasan, *Improved bounds and algorithms for hypergraph 2-coloring*, Random Struct. Algor. 16(1) (2000), 4–32.

[25] L. Duraj, G. Gutowski, and J. Kozik, *Chip games and paintability*, Electron. J. Combin. 23:3 (2016), Paper 3.3

[26] S. M. Ulam, *A collection of mathematical problems*, Interscience Tracts in Pure and Applied Mathematics 8 (Interscience Publishers, 1960).

[27] B. Manvel, *Some basic observations on Kelly's conjecture for graphs*, Discrete Math. 8 (1974), 181–185.

[28] R. P. Stanley, *Weyl groups, the hard Lefschetz theorem, and the Sperner property*, SIAM J. Algebraic and Discrete Methods 1 (1980), 168–184.

[29] Jacob Goodman under the pseudonym "Harry Dweighter" in American Math. Monthly 82 (1975), 1010.

[30] See `https://faculty.math.illinois.edu/~west/openp/`.

[31] See `http://www.openproblemgarden.org/`; the Open Problems Garden lists 227 problems in graph theory, 35 in combinatorics, and 160 in other areas.

[32] Suggested independently by Vizing, Albertson, Collins, Tucker, Gupta, and others, the conjecture apparently first appeared in print in B. Bollobás and A. J. Harris, *List-colourings of graphs*, Graphs Combin. 1:2 (1985), 115–127.

[33] J. D'Angelo and D. B. West, *Mathematical Thinking: Problem-Solving and Proofs*, 2nd edition, Prentice Hall, 2000.

Solving and Proofs[33]. What is your primary motivation for problem-solving? Were you interested in math competitions when you were a high school or college student?

West: I did not participate in any math competitions as a high school or college student. This is probably a good thing, since I have always been rather poorly educated as a mathematician and might have become quite discouraged by not being able to solve any of the questions.

I became interested in doing mathematics in a much different way. Rather than learning about problem-solving techniques across all of mathematics, I played with problems that interested me, using quite elementary methods. Later I learned a bit more, but there are still huge gaps in my mathematical background. Until I started teaching graduate courses I really did not know much about mathematical techniques other than induction.

Later, D'Angelo and I discovered that a guided approach to solving engagingly phrased problems is effective at drawing students into mathematics and getting them excited about learning more. This was our approach in *Mathematical Thinking*: use engaging problems that will interest students, but organize them according to the mathematical ideas involved. The goal is then to develop the mathematical principles in a coherent manner to solve the problems posed. Can one tile a big L-shaped region with small L-tiles (mathematical induction)? Does every year have a Friday the 13th (modular arithmetic)? Can any necklace with $2m$ gold beads and $2n$ silver beads be cut along some diagonal so that each of the two halves has half the beads of each color (Intermediate Value Theorem)? And so on...

The advantage of this over an earlier curriculum that began with calculus and then dumped students into courses in algebra or analysis is that the earlier approach required students to simultaneously absorb abstract ideas and techniques of mathematical proof. By using engaging problems as the applications, we aim to develop first the fundamentals of mathematical reasoning as a common background, in preparation for introducing abstract concepts in later courses. Here we can also focus on developing skills in communicating mathematics.

Mansour: You have advised around 40 Ph.D. students in their Ph.D. theses. How important is it to collaborate with Ph.D. students and pass on knowledge to them? Do you maintain contact with your students?

West: I maintained some level of contact after graduation with about two-thirds of my students, especially with those who remained in academia. Advising students and collaborating with them has been among the great joys of my career. I see the goal of the process as helping them develop their own style and approach to mathematics rather than passing on knowledge.

With this goal and because I had many students, I did not tell them to work on particular problems. Rather, we explored what interested the students and tried to develop interesting problems within those topics. We also gleaned research problems from listening to conference talks about new topics.

Passing on knowledge provides a base for students to develop their own mathematics, but more important to the process of mathematical maturation is to develop communication skills and clarity of writing. This is true at both the undergraduate and graduate levels. Students should be encouraged (required!) to communicate mathematics in both oral and written forms. This is how they learn whether they actually understand something, and it will greatly help them going forward. For me, finding the clearest way to present particular material in oral or written form is an enjoyable puzzle. I stress to students that it requires revision, revision, revision, preferably after waiting long enough to forget what one has written. Filling students' theses with red ink was the origin of my grammar page[34] on the web.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

West: Get involved in research as early as possible, preferably in a topic one is curious about. If a formal program is not available during the semester or summer, try to find a faculty member or fellow students to discuss problems with. When considering a research career, gauging one's reaction to problem-solving is more important than learning lots of mathematics. If

[34] `https://faculty.math.illinois.edu/~west/grammar.html`.

a student finds doing research painful, then a research career may not be a good idea. If working out examples and thinking about how to generalize them is exciting, then such a career may be fulfilling.

Mansour: You have been editor-in-chief of the journal Discrete Mathematics since 2007. During your tenure, the journal's impact factor significantly increased. What factors make a journal prestigious, influential, and long-lasting? What do you think about the role of journals in scientific development?

West: Journals have been especially effective in recording the development of mathematics and facilitating further advances. Some of the role of facilitating further advances has moved to preprint servers, where new results are now available more quickly. The refereeing process to check proofs now moves somewhat in parallel with the availability of the results.

Journal prestige rests on the quality of the results published and the reputations of the authors and the editorial boards, and these factors affect each other. When a research community reaches a rough consensus about what are the best journals in a particular subject, it is difficult for other journals to change that conclusion as long as the most-respected journals maintain the policies and quality that led to that status.

In the case of *Discrete Mathematics*, I believe that many factors influenced the improvement. Chief among these was building a board of Associate Editors consisting of well-respected experts in various subject areas, who could both raise the standards for accepted papers in their areas and attract better papers in those areas. Along with being more selective, we reduced the amount of papers we were publishing and stopped publishing issues that were mere conference proceedings. This allowed us to eliminate our publication backlog and improve response time, which encouraged more authors to consider us as a venue. Finally, the explosion of good-quality research without an explosion in top-quality journals improved the quality of all second-level journals.

Mansour: Professional mathematicians usually distribute their time among research, teaching, supervision, and service to the community (organizing conferences/workshops, being on the editorial boards of journals, writing reviews for papers or refereeing, etc.) You are very successful in all these areas. What is your secret?

West: Don't sleep. For most of my career before retirement I often slept about six hours per night during the week (sometimes less) in order to grade papers, prepare lectures, or try to finish proofs, catching up on sleep on the weekends. This probably is not a good approach for most people. Part of what enabled it to work for me is that all my children are academic. Also I have a very understanding and supportive spouse.

More useful advice is to spend time wisely. Academic departments should not expect all faculty to contribute equally in all activities. Individuals should contribute in the ways that they find rewarding and engaging. That makes it easier to give best effort. This approach may be unrealistic in small departments, where fewer people are available. I was fortunate to work in a very large department where I could happily put energy into aspects of the profession that I enjoyed and avoid some aspects of academic life that I would not be very good at, while still making a strong contribution to the mission of the department.

Mansour: Would you tell us about your interests besides mathematics?

West: I have had various hobbies over the years. I played squash, bridge, and go (weiqi) in the 1970s and 1980s, tried figure skating in the 1980s, sang in the Illinois Opera Theater chorus in the 1990s, danced salsa and did yardwork in the 1990s and 2000s (with my spouse, Ching Muyot), and studied Chinese in the 1970s and 2010s. In recent years I sang with an ukulele band, but now I mostly do puzzles: jigsaws, griddlers, Wordle, NYT spelling bee, and crossword puzzles from 28 years ago. I never cease to be amazed by the brilliance involved in creating clever crossword puzzles, which pile thematic coherence on top of constraints as restrictive as combinatorial designs.

Mansour: You gave talks at numerous conferences, workshops, and seminars. What do you think about the importance of such activities in a young researcher's career?

West: Early in my career I justified conference-going to my department head by explaining that I picked up many research problems there. Nowadays finding research problems is easier on the web, but conference activities remain valuable for young researchers in terms of building collaborations and providing exposure for the young researcher's results. Interactions outside lectures also provide opportunities to compare and contrast aspects of academic life at a variety of institutions. Some mathematicians prefer a solitary existence, but many find participating in a mathematical community more rewarding and supportive.

Mansour: Some of your research works cover *bar visibility number* associated with several families of graphs. Would you tell us about this notion more?

West: A *bar visibility representation*[35] of a graph G assigns each vertex a set of horizontal segments (bars) in the plane so that vertices u and v are adjacent in G if and only some bar assigned to u can "see" some bar assigned to v along an unobstructed vertical channel of positive width. The *bar visibility number* $b(G)$ is the minimum, over all bar visibility representations of G, of the maximum number of bars assigned to a single vertex. By representing edges separately, one can see that representations exist and that the maximum degree is a trivial upper bound on the bar visibility number. There is also a version for directed graphs in which a represented edge is oriented from the lower bar to the higher bar.

This notion arose by combining the idea of interval representations of graphs with the notion of the visibility graph of a family of objects in motion planning. In a *multi-interval representation*, the bars are all intervals along the real line, and adjacency occurs when some interval for one vertex intersects some interval for the other vertex. The *interval number*[36] $i(G)$ is the minimum, over all multi-interval representations of G, of the maximum number of intervals assigned to a single vertex. Both $b(G)$ and $i(G)$ are measures of the complexity of representing a graph.

Extremal bounds for various families of n-vertex graphs have been studied in the two models. Griggs[37] proved $i(G) \le \lceil (n+1)/4 \rceil$ when G has n vertices, achieved by the balanced complete bipartite graph. Cao, Yang, and I[38] determined $b(K_{\lceil n/2 \rceil, \lfloor n/2 \rceil})$ exactly; it is about half the value of the interval number. In contrast, for complete graphs $i(K_n) = 1$, but Chang, Hutchinson, Lehel, and I[35] showed $b(K_n) = \lceil n/6 \rceil$. It remains open whether this is the maximum over n-vertex graphs, though Feng, Yang, and I[39] proved $b(G) \le b(K_n) + 1$ when G has n vertices.

Mansour: We want to ask a question on your joint work with J. A. Noel, H. Wu, X. Zhu[40] related to *Ohba's Conjecture*. Would you tell us about your result and the essential ideas behind it? What are some possible future directions related to this conjecture?

West: Ohba's Conjecture concerned the choice number or list chromatic number of a graph, as defined in an earlier answer. The notation for choice number in computer science has mostly been $\mathrm{ch}(G)$, but mathematicians mostly have preferred $\chi_\ell(G)$, given the natural inequality with chromatic number: $\chi_\ell(G) \ge \chi(G)$. Even for $K_{\lceil n/2 \rceil, \lfloor n/2 \rceil}$, with chromatic number 2, the choice number grows (logarithmically) with n. Ohba conjectured[41] that when G has n vertices and chromatic number k, equality holds in $\chi_\ell(G) \ge \chi(G)$ as long as $n \le 2k+1$, which would be sharp. This conjecture was proved by Noel, Reed, and Wu[42].

Our later result explores what happens for larger n; for k-chromatic graphs with n vertices, we proved $\chi_\ell(G) \le \max\{k, \lceil (n+k-1)/3 \rceil\}$. This bound is sharp for $2k \le n \le 3k$, but it is not sharp beyond that. Noel conjectured that, among n-vertex k-chromatic graphs, the choice number is maximized by a complete k-partite graph with independence number $\lceil n/k \rceil$: in par-

[35] Y.-W. Chang, J. P. Hutchinson, M. S. Jacobson, J. Lehel, and D. B. West, *The bar visibility number of a graph*, SIAM J. Discrete Math. 18:3 (2004/05), 462–471.

[36] W. T. Trotter Jr. and F. Harary, *On double and multiple interval graphs*, J. Graph Theory 3:3 (1979), 205–211.

[37] J. R. Griggs, *Extremal values of the interval number of a graph. II.* Discrete Math. 28:1 (1979), 37–47.

[38] W. Cao, D. B. West, and Y. Yang, *On the bar visibility number of complete bipartite graphs*, SIAM J. Discrete Math. 35:3 (2021), 2234–2248.

[39] Y. Feng, D. B. West, and Y. Yang, *Upper bounds for bar visibility of subgraphs and n-vertex graphs*, Discrete Appl. Math. 283 (2020), 272–274.

[40] J. A. Noel, D. B. West, H. Wu, and X. Zhu, *Beyond Ohba's conjecture: a bound on the choice number of k-chromatic graphs with n vertices*, European J. Combin. 43 (2015), 295–305.

[41] K. Ohba, *On chromatic-choosable graphs*, J. Graph Theory 40:2 (2002), 130–135.

[42] J. A. Noel, B. A. Reed, and H. Wu, *A proof of a conjecture of Ohba*, J. Graph Theory 79:2 (2015), 86–102.

ticular, the well-known Turán graph. This would be the natural future direction, improving the bounds for choice number of n-vertex k-chromatic graphs for larger n.

In its essential ideas, our result relies heavily on the proof of Ohba's Conjecture by Noel, Reed, and Wu. The general approach is to restrict the properties of minimal counterexamples, where a counterexample would consist of a graph and an assignment of lists of size k to the vertices from which a proper coloring could not be chosen. This approach is promising also for Noel's conjecture.

Mansour: One of your areas of research is Extremal Combinatorics, a branch of combinatorial mathematics that studies how large or small a collection of finite objects (numbers, graphs, vectors, sets, etc.) can be under certain restrictions. Can you explain why it is crucial to understand the size of such collections? Would you give some examples of interesting applications of results from extremal combinatorics to other fields?

West: I think of Extremal Combinatorics in a broad sense as the analogue in discrete mathematics of worst-case complexity in computer science and constrained optimization in operations research. When we know that something can be done, it is natural to ask what is the best way to do it. Such results can lead to guarantees or restrictions on the performance of algorithms, for example.

One of my favorite applications of extremality is Andrew Yao's use[43] of Ramsey's Theorem (in 1981) to prove that the optimal way to store keys in a linear table to support membership queries is to store the keys in sorted order and use binary search.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

West: Once we know that a structure exists with certain properties, we can begin to ask how many of them there are. If we can describe them all, we can study additional properties they have. When we solve extremal problems, we can ask whether there is a unique extreme configuration or many; if the extremal configuration is unique or has special properties, then we can seek improved bounds when such configurations are forbidden. A famous example of this is the Erdős–Ko–Rado Theorem[44], where (with $k \leq n/2$) forbidding intersecting families of k-subsets of an n-set whose members all contain a fixed element leads to a significantly smaller bound[45] on the maximum size of an intersecting family.

Enumerative problems and techniques have not figured much in my research. In a recent paper with Ben Moore[46], we turned an old result about the existence of certain lengths of cycles in color-critical graphs into an enumerative result guaranteeing many such cycles. Another recent enumerative result, with Alexandr Kostochka and Zimu Xiang[47], gives lower bounds on the number of maximum matchings in certain bipartite graphs. To be honest, though, these are really extremal problems, seeking to find the minimum value of the number of such cycles or matchings over a family of graphs, which we express by saying that the lower bounds are sharp.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

West: Because the vast majority of my work is joint with students and colleagues, it is hard to recall the thought process for particular results. Usually there has been a succession of small insights by various participants that eventually led to a solution of the problem. It is gratifying also when further insights lead to simplification of proofs by understanding arguments in greater generality, which can happen while writing the paper.

In the spirit of your question, I will say that sitting for long hours puzzling over a particular point often is quite inefficient. The answer

[43] A. C. C. Yao, *Should tables be sorted?* J. Assoc. Comput. Mach. 28:3 (1981), 615–628.

[44] P. Erdős, C. Ko, and R. Rado, *Intersection theorems for systems of finite sets*, Quart. J. Math. Oxford Ser. (2) 12 (1961), 313–320.

[45] A. J. W. Hilton and E. C. Milner, *Some intersection theorems for systems of finite sets*, Quart. J. Math. Oxford Ser. (2) 18 (1967), 369–384.

[46] B. R. Moore and D. B. West, *Cycles in color-critical graphs*, Electron. J. Combin. 28:4 (2021), Paper No. 4.35.

[47] A. V. Kostochka, D. B. West, and Z. Xiang, *Sharp lower bounds for the number of maximum matchings in bipartite multigraphs*, arXiv:2211.10427.

feels so close that one does not want to give up and go to bed, but doing so can bring a new idea that unlocks the jam. The same principle applies when describing the difficulty to a student or colleague. The act of vocalizing the point instead of just thinking about it forces us to approach it in a different way, as do the questions asked by the listener, and this can unlock the secret.

I guess a lot of my papers do have stories behind them; I will mention the one on bar visibility number of bipartite graphs that you asked about earlier. The result was in my student Weiting Cao's thesis in 2006, but it was very difficult to understand the presentation. Weiting went off to industry and we did not publish the result. Much later, I encountered a preprint by Yan Yang giving a nice proof of a special case, and we joined forces. After ignoring the problem for nearly 15 years and studying Yan's illustrations of key examples, an elegant way to organize the cases and to simplify and explain Weiting's construction gradually evolved.

At the opposite end of the time scale, my paper with Martin Aigner[48] was conceived and worked out during an afternoon hike at Oberwolfach.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

West: There are many problems that I thought about over the years and would like to have solved but did not. I have written a number of papers on various aspects of graph reconstruction, which comes to mind because I am still actively thinking about this area. I am fond of saying that when mathematicians cannot solve a problem they define a harder problem, meaning a more general or more detailed problem. This is the way I always introduced graph Ramsey theory in lecture courses. Something similar has happened with reconstruction, where we consider added information (degree-associated reconstruction), the number of substructures needed to determine a graph (reconstruction number), reconstruction from smaller subgraphs (ℓ-reconstructibility), reconstruction of various graph invariants, the resolution of the conjecture on special families of graphs, and combinations of these various refinements. I cannot say I have made tremendous progress on any of these aspects, but I have enjoyed thinking about them and answering small pieces of the problems.

Mansour: In a very recent short article[49] published in the Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating the ethical aspects of the question "Who Owns the Theorem?" concluded that "Mathematical truths exist, and mathematicians only discover them." On the other side, there are opinions that "mathematical truths are invented". As a third way, some people claim that it is both invented and discovered. What do you think about this old discussion? More precisely, do you believe that you *invent* or *discover* your theorems?

West: In an ideal world, this is an unimportant question. However, now that efforts are made to "monetize" all kinds of things, such questions could become quite consequential. I think courts have held that mathematical facts cannot be patented, because they simply exist. I believe that we can invent mathematical structures, and similarly the definitions that we make of concepts are inventions, but the truths we prove about them are discoveries. This is perhaps only a semantic distinction, in that the inventor of a mathematical structure does not own it. On the other hand, an efficient algorithm encoded in software (such as Google's Pagerank), is an extremely valuable invention; this is applied mathematics.

Mansour: Professor Douglas West, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

West: You are welcome, and thank you for this opportunity to reflect and philosophize about some interesting aspects of mathematical life.

[48] M. Aigner and D. B. West, *Sorting by insertion of leading elements*, J. Combin. Theory Ser. A 45:2 (1987), 306–309.

[49] M. B. Nathanson, *Who Owns the Theorem?* The best writing on Mathematics 2021, Princeton: Princeton University Press, 2022, 255–257.