ecajournal.haifa.ac.il
Enumerative Combinatorics and Applications
ECA **4:2** (2024) Interview #S3I4
https://doi.org/10.54550/ECA2024V4S2I4

Interview with Alan Frieze

Toufik Mansour

Alan Frieze completed his Ph.D. at the University of London in 1975 under the supervision of Keith Wolfenden. His research interests include combinatorics, discrete optimization, and theoretical computer science. Honors and awards he has received include the Fulkerson Prize (jointly with Martin Dyer and Ravi Kannan) (1991), SIAM Fellow (2011), Fellow of the American Mathematical Society (2012), and the Simons Foundation Fellowship (2015). He was a plenary speaker at the 2014 International Congress of Mathematics. In 2022 he became Orion Hoch Professor of Mathematical Sciences at Carnegie Mellon University. Throughout his career he has served on the editorial boards of ten leading journals that include Combinatorics, Probability & Computing, the Journal of Combinatorial Optimization, and Random Structures & Algorithms.

Mansour: Professor Frieze, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Frieze: This is a difficult question. There probably is no definite answer. As an approximation, it is the study of discrete mathematical structures.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Frieze: This is a very positive development. The flow of ideas between different subjects can only help with the resolution of problems. For example, some people solved difficult problems using tools from algebra, geometry, topology, etc.

Mansour: What have been some of the main goals of your research?

Frieze: There is the selfish goal of getting the satisfaction of solving problems. I am also very interested in trying to explain why computational problems that are hard in the worst case can be solved efficiently in practice. I would also like to be able to create useful probabilistic models of complex societal and biological structures, for example, the brain.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Frieze: Mathematics was always my favorite subject. I had a very influential high school teacher who gave me some books to read. I remember being fascinated by "What is Mathematics?" by Courant and Robbins.

Mansour: Were there specific problems that made you first interested in combinatorics?

Frieze: I came at combinatorics through combinatorial optimization. I had a brief career in operations research after I finished a master's degree and I was intrigued by the Hungarian algorithm for the assignment problem.

Mansour: What was the reason you chose the University of London for your Ph.D. and your advisor Keith Wolfenden?

Frieze: I was living in London and I searched for someone to supervise research in Operations Research.

Mansour: What would guide you in your research? A general theoretical question or a

Toufik Mansour is a professor of mathematics at the University of Haifa, Israel. His email address is tmansour@univ.haifa.ac.il

specific problem?

Frieze: Keith gave me a survey paper and asked me to find something of interest. I started with work on the Simple Plant Location problem[1].

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Frieze: Most definitely yes. It helps to have a target.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Frieze: I can only speak for results that are related to my research. If we go back sixty years then there is the paper by Erdős and Rényi[2] on the evolution of random graphs. If we go back forty years then there is the paper by Shamir and Spencer[3] that brought martingale inequalities into our field. More recently, there are the papers on the Container Method by Balogh, Morris, and Samotij[4] and by Saxton and Thomason[5]. Then I would mention the papers by Frankston, Kahn, Narayanan, and Park[6] and by Park and Pham[7] on thresholds. Finally, we have the breakthrough results on Ramsey numbers: $R(4,t)$ by Matheus and Verstraete[8] and $R(k,k)$ by Campos, Griffiths, Morris, and Sahasrabudhe[9]. (I have no results on Ramsey theory, but I have thought often about improving the lower bound on $R(k,k)$.)

Mansour: What are the top three open questions in your list?

Frieze: My list is very narrow and maybe not very ambitious. I would like to prove that $G_{n,cn}$ conditioned on minimum degree three is Hamiltonian with high probability. Then I would like to determine the likely cover-time of two-dimensional random geometric graphs. These I think are within my capabilities. A more ambitious problem would be that of improving Spencer's lower bound on diagonal Ramsey numbers[10].

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Frieze: To me this is a matter of taste. On the other hand, we should all know the basics of algebra, analysis, and probability.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Frieze: I don't see the point of labeling some areas as pure and some as applied. What is important is that sometimes mathematics is used to solve non-mathematical problems.

Mansour: As an advisor, you have influenced the careers of many students. What advice do you have for young mathematicians who are just starting their academic journeys?

Frieze: First of all find an area that interests you and where you think you can make a significant contribution. Then have a backup plan in case things do not work out. There is a lot of competition for getting onto the first rung of an academic career.

Mansour: Jointly with Martin Dyer and Ravindran Kannan you were awarded the *Fulkerson Prize in Discrete Mathematics* by the American Mathematical Society and the Mathematical Programming Society for your paper *A random polynomial time algorithm for approximating the volume of convex bodies*[11]. Can you highlight the main ideas behind this work?

Frieze: The main idea was to reduce the problem of estimating the volume of a convex body K to that of sampling uniformly from K. About that time Broder and then Jerrum and

[1]A. M. Frieze, *A cost function property for plant location problems*, Math. Programming 7 (1974), 245–248.

[2]P. Erdös and A. Rényi, *On the evolution of random graphs*, Publ. Math. Inst. Hungar. Acad. Sci. 5 (1960), 17–61.

[3]E. Shamir and J. Spencer, *Sharp concentration of the chromatic number of random graphs $G_{n,p}$*, Combinatorica 7 (1987), 121–129.

[4]J. Balogh, R. Morris, and W. Samotij, *Independent sets in hypergraphs*, J. AMS 28:3 (2015), 669–709.

[5]D. Saxton and A. Thomason, *Online containers for hypergraphs, with applications to linear equations*, J. Combin. Theory, Ser. B 121 (2016), 248–283.

[6]K. Frankston, J. Kahn, B. Narayanan, and J. Park, *Thresholds versus fractional expectation thresholds*, Ann. of Math. 194 (2021), 475–495.

[7]J. Park and H. Pham, *A Proof of the Kahn-Kalai Conjecture*, 2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS), Denver, CO, USA, 2022, 636–639.

[8]S. Matheus and J. Verstraete, *The asymptotics of $r(4,t)$*, Ann. of Math., to appear.

[9]M. Campos, S. Griffiths, R. Morris, and J. Sahasrabuhde, *An exponential improvement for diagonal Ramsey*, arXiv:2303.09521.

[10]J. Spencer, *Ramsey's theorem – A new lower bound*, J. Combin. Theory, Ser. A 18:1 (1975), 108–115.

[11]M. E. Dyer, A. M. Frieze, and R. Kannan, *A random polynomial time algorithm for approximating the volume of convex bodies*, STOC (1989), 375–381

Sinclair had just shown the power of using a Markov chain to sample from a complex distribution. So we set up a Markov chain (random walk) that would enable us to efficiently sample from K. Since then, there have been many great improvements in the implementation of this idea.

Mansour: *Hamilton cycles* are a recurring theme in many of your research articles. Could you explain the significance of these cycles in Probabilistic Combinatorics and the broader context of your work?

Frieze: I just think that Hamilton cycles are a rather natural object to study and they have properties that are very exploitable in the case of random graphs and hypergraphs.

Mansour: In one of your papers *A scaling limit for the length of the longest cycle in a sparse random graph*[12], coauthored with Michael Anastos, you considered the length $L_{c,n}$ of the longest cycle in a sparse random graph $G_{n,p}, p = c/n$, c-constant, and showed that for large c there exists a function $f(c)$ such that $L_{c,n}/n \to f(c)$ a.s. Would you say a few lines about this work?

Frieze: The basic idea is to separate the vertices of the graph into two sets A and B where the minimum degree in B is high. Then optimally cover as many vertices in A with paths as possible. Then find a cycle that contains B as well as the previously constructed paths.

Mansour: The P versus NP problem is a major unsolved problem in theoretical computer science. Many researchers claim that if P=NP, then the world would be a profoundly different place than we usually assume. What do you think about this problem and such claims? Do you expect to see a solution for it in the near future?

Frieze: I agree that P=NP would say something very surprising about computation. I don't expect the P versus NP question to be resolved very soon.

Mansour: Is there a result in graph theory that you consider surprising or unintuitive?

Frieze: I think that the Perfect Graph Theorem is rather surprising, but maybe I don't know enough about the problem.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Frieze: My work is all about approximately counting various structures. Either we want to show that almost all combinatorial structures have a certain property or that almost none of them have it.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Frieze: One of my favorite questions is about Hamilton cycles in the random graph G_{k-out}[13]. In this graph, each vertex independently chooses k random neighbors. I got interested because David Walkup[14] had produced a very nice result on perfect matchings in a bipartite version of the problem. Trevor Fenner and I worked on the Hamiltonicity question for some time and I remember making a breakthrough on the question between Stepney Green underground and Mile End underground on my way to work one morning. I realized that a certain double-counting argument would break the problem. This gave the result for $k = 23$ and then after a series of papers[15,16,17] and about 25 years, Tom Bohman[18] and I got the right answer, G_{3-out} is Hamiltonian with high probability.

Mansour: You have served on the editorial board of many prestigious journals. What features distinguish a prestigious one from the others? What do you think about the importance of high-quality journals in mathematical research?

Frieze: With the Arxiv, good journals are less

[12]M. Anastos and A. Frieze, *A scaling limit for the length of the longest cycle in a sparse random graph*, J. Combin. Theory, Ser. B 148 (2021), 184–208.

[13]T. Fenner and A. M. Frieze, *On the existence of Hamiltonian cycles in a class of random graphs*, Discrete Math. 45 (1983), 301-305.

[14]D. W. Walkup, *Matchings in random regular bipartite digraphs*, Discrete Math. 31:1 (1980), 59–64.

[15]C. Cooper and A. M. Frieze, *Hamilton cycles in random graphs and directed graphs*, Random Struct. Algorithms 16 (2000), 368–401.

[16]A. M. Frieze,*Finding Hamilton cycles in sparse random graphs*, J. Combin. Theory, Ser. B 44:2 (1988), 230–250.

[17]A. M. Frieze and T. Luczak, *Hamiltonian cycles in a class of random graphs: one step further*, n Proceedings of Random Graphs '87, Edited by M. Karonski, J. Jaworski, and A. Rucinski, John Wiley and Sons (1989), 53–59.

[18]T. Bohman and A. M. Frieze, *Hamilton cycles in 3-out*, Random Struct. Algorithms 35 (2009), 393–417.

and less important as a means of spreading knowledge. They are however important in verifying claims and keeping the subject honest.

Mansour: Is artificial intelligence a cure or a curse for humanity[19]? The visionary historian Yuval Noah Hariri from the Department of History at the Hebrew University of Jerusalem claims, "Homo sapiens as we know them will disappear in a century or so." What do you think?

Frieze: Predicting 100 years into the future is very bold. Who knows what the world will be like?

Mansour: Professor Frieze, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

Frieze: It has been my pleasure to answer your questions.

[19]See `https://www.theguardian.com/culture/2017/mar/19/yuval-harari-sapiens-readers-questions-lucy-prebble-arianna-huffington-future-of-humanity`